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New hybrid model of dynamic rail track response in the vertical direction

Abstract: This article presents a new hybrid model for analysing the dynamics of a railway track subjected to excitations generated by a rail vehicle, taking into account the dynamic forces at the wheel-rail interface. It is demonstrated that combining a multi-body force generator with a continuous multi-layer analytical model of the railway track is feasible and enables the analysis of both time-varying forces generated by the vehicle and the response of the track surface depending on the values of the mechanical parameters of the structure. The model of mechanical vibrations of the track in the vertical direction is solved using a semi-analytical approximation method using wavelet filters. Combined with a force description obtained by modelling the mass system corresponding to the vehicle, this represents a novel approach to the parametric analysis of track response.

Keywords: rail track; rail vehicle; wheel-rail contact forces; hybrid model

Introduction

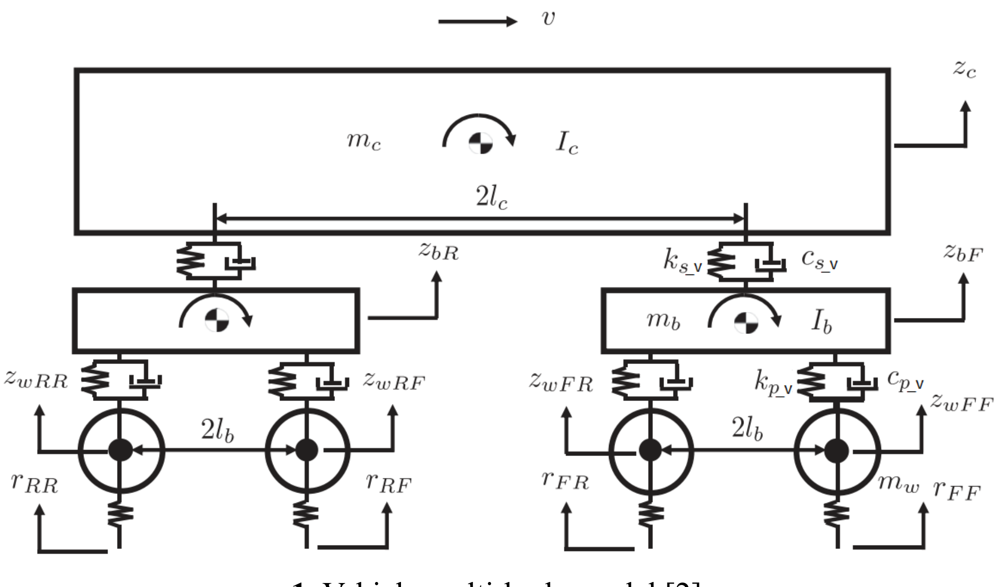
The paper presents a new model of dynamic track response in the vertical direction. The model consists of two independent parts describing vehicle and rail track. The vehicle model is used to analyse the dynamic behaviour of a train when running on a track with irregularities and to determine the dynamic forces generated at the wheel-rail contact. The dynamic response of the track system under load from the rolling stock is analysed then by using analytical multilayer continuous model of rail track.

Usually two factors of dynamic interaction between vehicle and rail track are considered, the weight of vehicle producing static or quasi-static response and periodical irregularities in rail track, leading to fast changing in time characteristics of the response. The additional sources of dynamic interactions from the rolling stock are rail and wheels imperfections, which cause the mechanical system of the train to be forced out of equilibrium. In a hypothetical case of a train travelling with perfectly round wheels on a perfectly rigid and flat surface, one deals with quasi-static interaction (a moving force of constant value) – there would be no changing in time impacts. Movement on an irregular track, which additionally behaves elastically under the influence of a moving force, or bending rails, are effects that cause particular masses of the train's mechanical system to be thrown off balance. Thus, the passage of a train generates fast changing in time dynamic influence. Factors that are important for the magnitude of the common interactions and its dynamic characteristics include among others: train weight, first and second degree stiffness with accompanying damping, train speed, imperfections in the train wheels, rail irregularities, damping properties or vehicle axles configuration. This article presents an efficient method for determining the dynamic forces generated at wheel-rail contact. The forces determined are then used in the analysis of the dynamic response of the track in the vertical direction. The calculations of multi-body dynamics for train vehicles presented in this study are well recognized in the scientific literature [e.g. 1, 2].

The lower part of the model is an analytical description of the railway track, represented as a continuous multi-layer system, in which the rails are described as beams of infinite length. The sleepers constitute a rigid body within the second track layer, connected to the rails by a viscoelastic layer, representing the rail fastening system. The entire structure rests on a viscoelastic foundation, whose parameters can be described according to Zimmermann's theory, taking into account the various subgrade layers. This model can be solved using a semi-analytical method based on wavelet approximations, opening the way to nonlinear analysis, crucial for the actual mechanical parameters of the rail fastening system or for highly degraded subgrades.

1. Vehicle model

The vehicle model is considered as a half of a railcar, in which the force is calculated per rail. The model consists of a car body, two bogies, and four wheels. The train wheels are modelled as masses with one degree of freedom (vertical displacement). The bogies and car body are modelled as masses with a moment of inertia and two degrees of freedom (vertical displacement and rotation). The connection between the wheels and the bogies is the primary suspension, modelled by Kelvin-Voight elements with stiffness and damping characteristics. The bogies are connected to the car body by a secondary suspension, also modelled as Kelvin-Voight elements. The model diagram is shown in Figure **1**.



1. Vehicle multi-body model [2]

The general equation of motion for the system is expressed as:

$$[\mathbf{M}]\{\ddot{\boldsymbol{u}}\} + [\mathbf{C}]\{\dot{\boldsymbol{u}}\} + [\mathbf{K}]\{\mathbf{u}\} = \{\mathbf{F}\} \quad (1)$$

where: M – mass matrix, C – damping matrix, K – stiffness matrix, u – displacement, $\dot{u}$ – velocity, $\ddot{u}$ – acceleration, F – external load (desired dynamic load generated by the system).

The following notation is used:
$r_{FF},\ r_{FR},\ r_{RF},\ r_{RR}$ [m] – track irregularities under the wheels;
$z_{wFF},\ z_{wFR},\ z_{wRF},\ z_{wRR}$ [m] – vertical displacement of the wheels;
$z_{bF},\ z_{bR}$ [m] – vertical displacement of the bogies;
z_c [m] – vertical displacement of the car body;
m_w [kg] – single wheel mass;
m_b [kg] – single bogie mass;
m_c [kg] – car body mass;
I_b [kg·m^2] – moment of inertia of single bogie;
I_c [kg·m^2] – moment of inertia of car body;
k_{p_v} [N/m] – primary suspension stiffness;
c_{p_v} [N·s/m] – primary suspension damping;
k_{s_v} [N/m] – secondary suspension stiffness;
c_{s_v} [N·s/m] – secondary suspension damping;
$l_b,\ l_c$ [m] – spacing of wheels and bogies;
V [m/s] – speed of the train;

The matrices and vectors used in Eq. (1) are expressed as follows: the mass matrix (2), the stiffness matrix (3), the damping matrix (4), and the generalized displacement vector describing the vertical displacements and rotational angles of the individual vehicle components (5).

$$\mathbf{M} = \begin{bmatrix} m_c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I_c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_b & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_b & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_b & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_b & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_w & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_w & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_w & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_w \end{bmatrix} \tag{2}$$

$$\mathbf{K} = \begin{bmatrix} 2k_{s_v} & 0 & -k_{s_v} & 0 & -k_{s_v} & 0 & 0 & 0 & 0 & 0 \\ 0 & 2k_{s_v}l_c^2 & k_{s_v}l_c & 0 & -k_{s_v}l_c & 0 & 0 & 0 & 0 & 0 \\ -k_{s_v} & k_{s_v}l_c & 2k_{p_v}+k_{s_v} & 0 & 0 & 0 & -k_{p_v} & -k_{p_v} & 0 & 0 \\ 0 & 0 & 0 & 2k_{p_v}l_b^2 & 0 & 0 & l_bk_{p_v} & -l_bk_{p_v} & 0 & 0 \\ -k_{s_v} & -k_{s_v}l_c & 0 & 0 & 2k_{p_v}+k_{s_v} & 0 & 0 & 0 & -k_{p_v} & -k_{p_v} \\ 0 & 0 & 0 & 0 & 0 & 2k_{p_v}l_b^2 & 0 & 0 & l_bk_{p_v} & -l_bk_{p_v} \\ 0 & 0 & -k_{p_v} & l_bk_{p_v} & 0 & 0 & k_{p_v} & 0 & 0 & 0 \\ 0 & 0 & -k_{p_v} & -l_bk_{p_v} & 0 & 0 & 0 & k_{p_v} & 0 & 0 \\ 0 & 0 & 0 & 0 & -k_{p_v} & l_bk_{p_v} & 0 & 0 & k_{p_v} & 0 \\ 0 & 0 & 0 & 0 & -k_{p_v} & -l_bk_{p_v} & 0 & 0 & 0 & k_{p_v} \end{bmatrix} \tag{3}$$

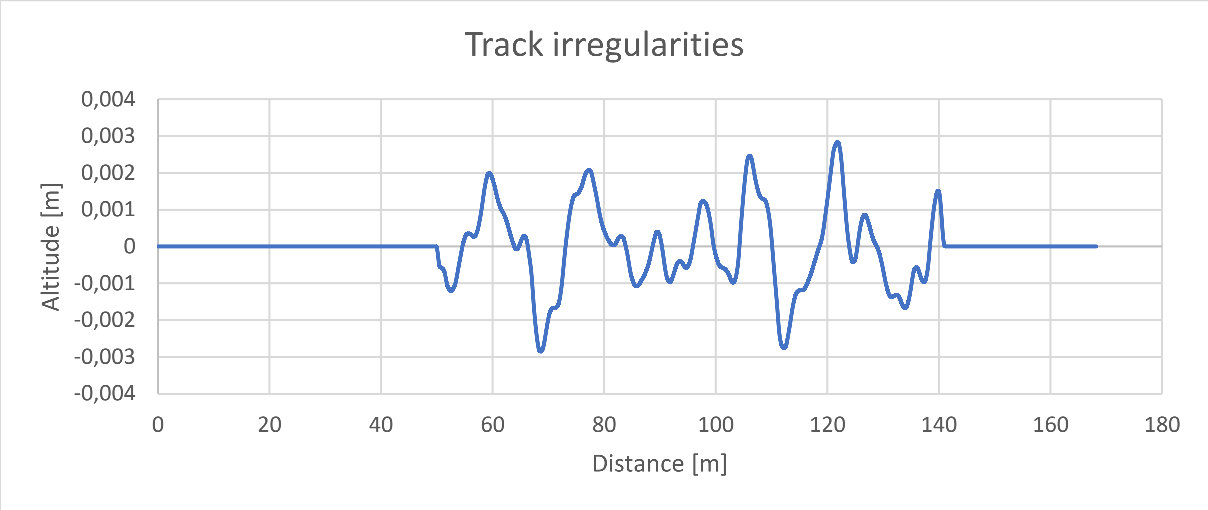
$$\mathbf{C} = \begin{bmatrix} 2c_{s_v} & 0 & -c_{s_v} & 0 & -c_{s_v} & 0 & 0 & 0 & 0 & 0 \\ 0 & 2c_{s_v}l_c^2 & c_{s_v}l_c & 0 & -c_{s_v}l_c & 0 & 0 & 0 & 0 & 0 \\ -c_{s_v} & c_{s_v}l_c & 2c_{p_v}+c_{s_v} & 0 & 0 & 0 & -c_{p_v} & -c_{p_v} & 0 & 0 \\ 0 & 0 & 0 & 2c_{p_v}l_b^2 & 0 & 0 & l_bc_{p_v} & -l_bc_{p_v} & 0 & 0 \\ -c_{s_v} & -c_{s_v}l_c & 0 & 0 & 2c_{p_v}+c_{s_v} & 0 & 0 & 0 & -c_{p_v} & -c_{p_v} \\ 0 & 0 & 0 & 0 & 0 & 2c_{p_v}l_b^2 & 0 & 0 & l_bc_{p_v} & -l_bc_{p_v} \\ 0 & 0 & -c_{p_v} & l_bc_{p_v} & 0 & 0 & c_{p_v} & 0 & 0 & 0 \\ 0 & 0 & -c_{p_v} & -l_bc_{p_v} & 0 & 0 & 0 & c_{p_v} & 0 & 0 \\ 0 & 0 & 0 & 0 & -c_{p_v} & l_bc_{p_v} & 0 & 0 & c_{p_v} & 0 \\ 0 & 0 & 0 & 0 & -c_{p_v} & -l_bc_{p_v} & 0 & 0 & 0 & c_{p_v} \end{bmatrix} \quad (4)$$

$$\mathbf{u} = \begin{bmatrix} z_c \\ \varphi_c \\ z_{bF} \\ \varphi_{bF} \\ z_{bR} \\ \varphi_{bR} \\ z_{wFF} \\ z_{wFR} \\ z_{wRF} \\ z_{wRR} \end{bmatrix} \quad (5)$$

The system at a stationary state generates quasi-static pressure on the track with values resulting from its weight. When the system moves along geometrically irregular track, individual masses are displaced from their equilibrium positions, hence generating dynamic forces. The track containing irregularities along which the vehicle moves forms the excitation for the whole system. The calculations are performed using the exemplary track irregularity profile shown in Figure **2**. The track irregularity profile was taken from publication [3]. The data used in the analysis were approximated from the plot provided in the referenced work.

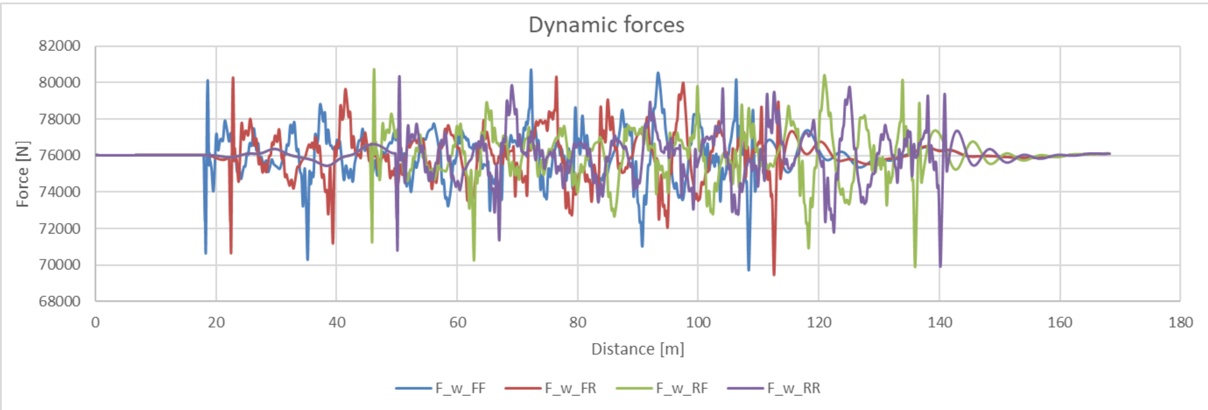
As noted in the paper, the profile was generated from the power spectral density and maximum amplitudes used are the limiting amplitudes of rail irregularities with wavelengths of 3–25 m.

Subsequently, the profile showed in Figure **2** was extended by adding additional “flat” segments on both sides, ensuring that all wheels of the vehicle start on an undeformed track and that every wheel passes through the entire irregularity profile.



2. Exemplary geometry of track irregularities

The calculations are performed for a train running on a given track at speeds of 115 km/h and 250 km/h. The system is solved numerically using the Newmark method. The interaction between the train wheels and the rail is implemented using a nonlinear Hertz contact model [4]. Results of the dynamic response of the train wheels calculated for a speed of 115 km/h are shown in Figure **3**. This figure presents the dynamic forces generated by each of the four wheels of the railway vehicle (labeled as $F_{w,FF}$, $F_{w,FR}$, $F_{w,RF}$, $F_{w,RR}$, following the order shown in Fig. 1). The forces are plotted with respect to the longitudinal position of each wheel along the analysed track. The relative horizontal shifts between the individual force curves result from the actual wheelbase geometry of the vehicle.



3. Simulated dynamic forces

The results are obtained by numerically solving the multi-body dynamics system, which is a well-established method in the scientific community [e.g., 1, 2]. Based on the calculations, a high sensitivity of the numerical system is observed, among other things, to the time step of the analysis (it must be appropriately selected for the speed of the moving train and the profile of the irregularities). The way in which the irregularity is applied is also very important (even small changes in the vertical amplitude of the track irregularity, but applied in a jumping way, without a smooth transition, can lead to large force peaks).

2. Rail track model

The rail track model used for the calculations is based on an analytical approach. It describes a continuous multi-layer system, in which the leading component is the so-called double beam concept (Eq. 2), which accounts for the nonlinear stiffness of the layer between the beams and the foundation of the system [5, 6]:

$$EI_r \frac{\partial^4 u}{\partial x^4} + m_r \frac{\partial^2 u}{\partial t^2} + c_r \left(\frac{\partial u}{\partial t} - \frac{\partial w}{\partial t}\right) + k_r(u - w) + k_{Nr}u^3 - k_{Ns}w^3 = P(x,t)$$

$$EI_s \frac{\partial^4 w}{\partial x^4} + m_s \frac{\partial^2 w}{\partial t^2} + c_s \frac{\partial w}{\partial t} + k_s w - c_r \left(\frac{\partial u}{\partial t} - \frac{\partial w}{\partial t}\right) - k_r(u - w) + k_{Ns}w^3 - k_{Nr}u^3 = 0 \quad (6)$$

The following notation is used:
$u(x,t)$ [m] – vertical vibrations of rails;
EI_r [Nm^2] – bending stiffness of rail steel;
m_r [kg/m] – unit mass of rail;
k_r [N/m^2] – linear stiffness of the layer between rails and sleepers (including fastening system);
c_r [Ns/m^2] – viscous damping of the layer between rails and sleepers (including fastening system);
k_{Nr} [N/m^4] – nonlinear part of stiffness of the layer between rails and sleepers (including fastening system);
$w(x,t)$ [m] – vertical vibrations of sleepers;
EI_s [Nm^2] – bending stiffness of sleepers layer;
m_s [kg/m] – unit mass of sleepers;
k_s [N/m^2] – linear stiffness of the rail track foundation;
c_s [Ns/m^2] – viscous damping of the rail track foundation;
k_{Ns} [N/m^4] – nonlinear part of stiffness of the rail track foundation;
$P(x,t)$ [N/m] – a set of loads generated by axles of train moving uniformly along rails with constant speed V [m/s].

Equation 6 can describe the track dynamics for a ballastless rail track, and after removing the first term from the second line (EI_s), responsible for the bending stiffness, it provides an excellent and validated description of the dynamic response of a classical ballasted track [7]. In this case, dynamic changes in track response related to periodic track irregularities (such as additional rail deflections between sleepers) can be included in the load description. Generally, a load generated by a single wheel consists of 3 components: constant in time static part P_S generated by the weight of train, changing in time part P_D associated with periodic properties of stiffness and the part P_R responsible for other random factors influencing the load:

$$Q_k(x,t) = P_S(x,t) + P_D(x,t) + P_R(x,t),\ P(\tilde{x},t) = \sum_{k=0}^{L-1} Q_k(x,t) \quad (7)$$

Taking into account the force distribution in the wheel-rail contact area and the axles configuration, the single axle load can be described as [7, 8]:

$$Q_k(x,t) = (P_0 + \Delta P \cdot exp\ (i(\Omega_k t + \varphi_l)))\frac{1}{2a} cos^2\left(\frac{\pi(x - Vt - s_k)}{2a}\right) H(a^2 - (x - Vt - s_k)^2) \quad (8)$$

where:
Ω_k – frequency of axle force;
s_k – distances between forces (vehicle axles);
φ_l – angular frequency associated with wheel position on sleepers;
V – train speed;
$H(*)$ – Heaviside function used to describe the wheel-rail contact area;

ΔP – additional force generated by regular imperfections (e.g. deflection between sleepers).

Regardless of the form and complexity of the load (Eqs. 7 and 8), the system (6) can be solved using the Adomian's decomposition with respect to nonlinear factors (stiffness of the foundation and stiffness of the fastening system [8, 9]) and wavelet filter-based Fourier transform approximations [5-7, 10].

3. Computational examples

The following ballasted rail track parameters are used in computational examples [7]:

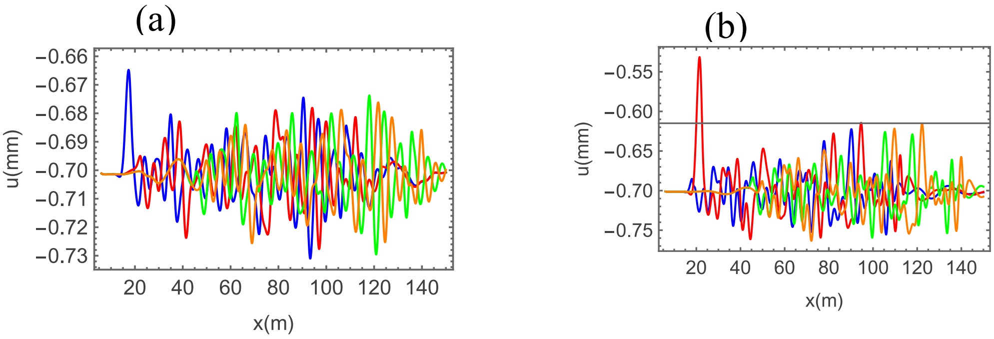
1. Rail type 60E1: Young's modulus E=2.1*10^8kN/m^2; moment of inertia in the vertical plane I_r =3055*10^{-8} m^4; unit mass m_r =60 kg/m;
2. Rail foundation (fastening system): k_r =8.8*10^7 N/m^2, c_r= 3950 Ns/m^2;
3. Rail track foundation: k_s =8.5*10^7 N/m^2 (k_s =2.8*10^7 N/m^2), c_s= 81*10^3 Ns/m^2, $k_{Ns} = 0; k_{Nr} = 0$;
4. Unit mass of sleeper: m_s =267 kg/m;
5. Train speed: V=115 km/h, V=250 km/h.

The component P_D is neglected and only contact dynamic forces generated by the system described in Chapter 1 are considered.
Vehicle parameters used in computational examples are presented in Table **1**. Description of the symbols used is provided in Chapter 1.

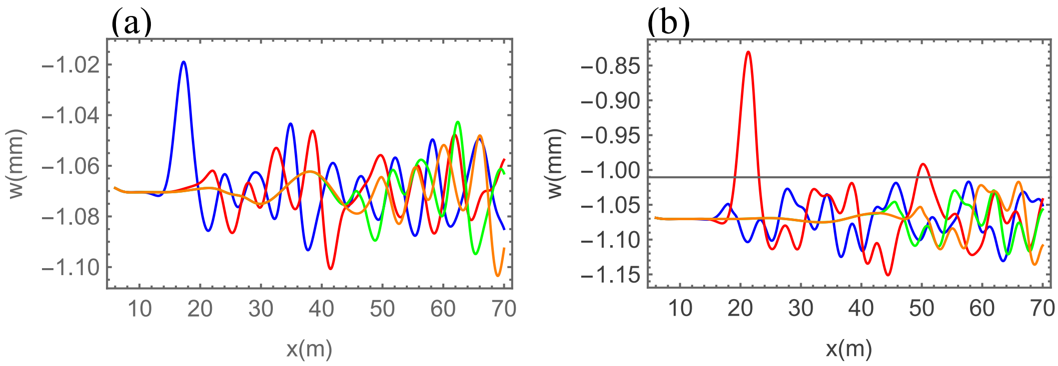
Tab. 1: Vehicle model parameters summary

parameter	**value**	**unit**
m_c	47800	kg
m_b	3500	kg
m_w	1800	kg
k_{p_v}	1200	kN/m
k_{s_v}	350	kN/m
c_{p_v}	10	kN*s
c_{s_v}	20	kN*s
I_c	694121	kg/m^2
I_b	8035	kg/m^2
l_c	13.8	m
l_b	2.1	m

The forces generated at the wheel-rail interface are implemented in a continuous two-layer analytical model of the rail track, described by equations (6), where the bending stiffness in the second equation is assumed to be zero, corresponding to the ballasted track parameters. However, the removal of the P_D component eliminates the periodicity of the considered structure. The influence of nonlinearities in the subgrade and the fastening system is also neglected.



4. Vertical displacement of rails for the random rail track section with good condition of rail track foundation (k_s =8.5*10^7 N/m^2) and two bogies (front-front – blue, front-rear – red, rear-front – green, rear-rear – orange) running with the speed of: (a) V=115 km/h; (b) V=250 km/h.



5. Vertical displacement of sleepers layer for the random rail track section with degraded rail track foundation (k_s =2.8*10^7 N/m^2) and two bogies (front-front – blue, front-rear – red, rear-front – green, rear-rear – orange) running with the speed of: (a) V=115 km/h; (b) V=250 km/h.

The examples are intended to demonstrate the possibility of combining hybrid multi-body techniques with a validated analytical model of the rail track, whose solution is based on semi-analytical wavelet approximations. This combination allows for a detailed analysis of the rail track response with respect to a number of parameters describing the vehicle in detail, but also taking into account the mechanical properties of the rail track and its foundation. Figures **4** and **5** show vertical vibrations considered for a number of varying physical and mechanical parameters, demonstrating the significant impact of both rail track foundation quality and vehicle speed on the track response. A detailed analysis requires further simulation work and is therefore a future project. Of particular importance may be the consideration of nonlinear system characteristics (see Eq. 6), the analysis of which in the considered configuration requires significant computational power and remains an open problem. The next step in the planned work is to develop a numerical model of the rail track and verify it using the hybrid system presented in this paper. This will allow for the consideration of a wider range of possible cases occurring in operating systems.

Conclusions

It is shown that it is possible to effectively combine a multi-body modelling approach for train-generated dynamic forces with analytical railway track models solved by using semi-analytical techniques. Such a model can be used to analyse both the wheel-rail contact forces generated by the vehicle and their impact on rail track dynamics, depending on the rail track foundation conditions and physical parameters of the system.

Source materials

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