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The moving chord method for determining the curvature of the track axis

Abstract: The paper adresses the issue of determining the curvature of the axis of the railway track in order to enable the determination of the geometrical characteristics of the measured route. A new concept of determining the curvature was used by the method of changing the angles of inclination of a moving (virtual) chord based on the knowlege of the Cartesian coordinates of a given region of the route. The verification of the proposed method, carried out on unambiguosly defined model geometric track layouts, showed the complete agreement of the obtained curvature diagrams with the diagrams constituting the basis for obtaining the corresponding geometric solution. The use of the moving chord method to determine the curvature of the operated railway track showed that the obtained curvature plots clearly differ from those for model layouts; they have a less regular, oscillating character, which results from the track deformation and measurement error. However, this does not prevent the possibility of estimating the basic geometrical parameters of the measured layout on this basis.

Keywords: railroad; curvature of the track axis; verification of a new calculation method; determination of application possibilities

Introduction

Geodetic measurements used to determine and evaluate the shape of the track axis are the basis for determining the basic geometric parameters of the route. In relation to the horizontal plane, these parameters are:

- location and length of straight sections,
- location of circular arcs along with determining their radius and length,
- location of the transition curves along with their type and length.

Based on this data, it becomes possible to determine the maximum speed of trains and - which seems to be a fundamental matter - to obtain data for designing the regulation of the track axis.

The measurement methods used have a very long tradition and, although they are subject to various innovations, they are characterized by high labor consumption and the related need to incur significant financial outlays. The rules for carrying out measurements are similar in different railway administrations [1-4, 10-12, 14]. In accordance with the regulations in force in Poland [13], straight sections are measured using the ordinate and abscissa method along the measurement lines, using the appropriate geodetic control network. Measurements on the arc include the measurement of horizontal arrows related to the chord determined by the theodolite's axis of sight (the so-called direct method) or the track axis adjustment marks (indirect method).

A more modern measuring system is a total station placed on a trolley, which additionally allows one to measure the inclination of the track (using an inclinometer) and measure its width. A radical improvement of the existing situation should be ensured by the mobile satellite measurement method developed in Poland for several years [15]. It consists of the continuous recording of track axis coordinates using GNSS receivers installed on a moving measurement platform and the use of the collected measurement data in appropriate calculation algorithms.

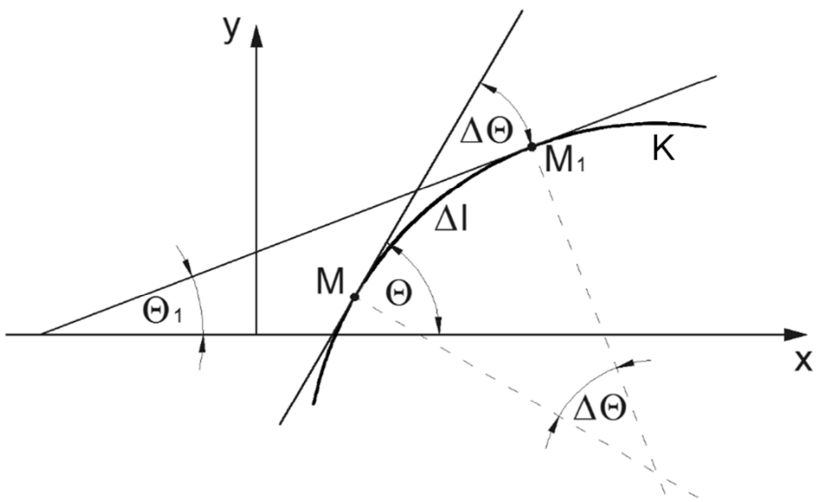
When identifying the measured geometrical system based on the measurements carried out, the most obvious solution is to determine the existing curvature. In engineering practice, the identification of the character of the horizontal curvature occurring in a given geometric layout of the track is most often carried out indirectly - based on the measured arrows from the chord stretched along the track. The arrow graph method is still very popular on railways, as there is virtually no other alternative to it. Since the arrow plot is very similar to the curvature plot, some people use this method as a way to determine the curvature of the track. From a formal point of view, this is of course unjustified. In addition, it should be taken into account that the graph of arrows shows the values of the measured horizontal arrows, but it does not specify the directions in which these arrows are measured. The reference line here is the directions of the chord settings, which are constantly changing. It should be noted that the measurement of arrows (horizontal and vertical) has been the basis for many years of diagnostic methods relating to the assessment of the geometric condition of railway tracks.

This paper presents a new method of curvature determination based on the knowledge of the Cartesian coordinates of the railway track axes. This method is based on the use of the change in the angle of inclination of the chord along the length of the geometric system. It has been referred to as the "moving chord method".

The idea of the moving chord method

The curvature of the track is defined over its length *l,* in a linear coordinate system. The measure of the curvature of the route is the ratio of the angle by which the direction of the longitudinal axis of the vehicle changes after passing a certain arc to the length of this arc (Fig. **1**). The curvature of the curve *K* at the point *M* is the limit approached by the ratio of the acute angle $\Delta\Theta$ included between the tangents to the curve *K* at points *M* and M_1 to the length of the arc Δl , when the point M_1 follows the curve *K* to the point *M*

$$\kappa = \lim_{\Delta l \to \infty} \left| \frac{\Delta\Theta}{\Delta l} \right| \tag{1}$$



1. Schematic diagram to explain the concept of curvature

If the analytic notation of the function is known , then the formula holds

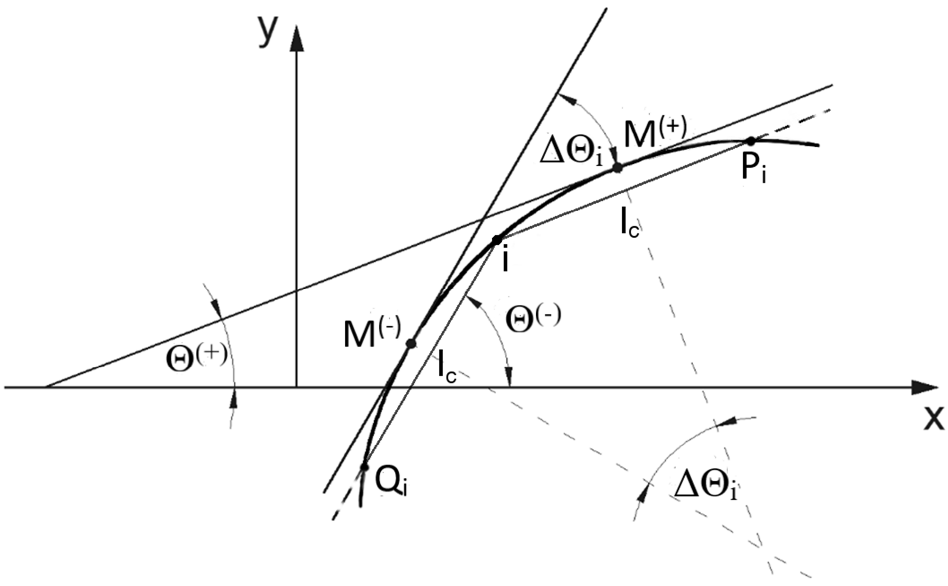
$$\kappa(l) = \frac{d}{dl}\Theta(l) \tag{2}$$

A practical way of determining the curvature (for small values of) is to use the simplified formula:

$$\kappa(l) \cong \frac{\Delta\Theta}{\Delta l} \tag{3}$$

Designing a geometric system, however, requires operating in a Cartesian coordinate system. As a result of this process, the coordinates of the track axis are determined, allowing it to be set out in the field. Determining the curvature of a given geometric system is therefore difficult because the direct use of formulas from a linear system is impossible.

In addition, the definition of curvature implies the need to operate with the angles of inclination of the tangent to the geometric system. If you have an analytical record of a given curve, this is of course not a problem. However, in a real railway track, most often deformed as a result of an operation, it is very difficult to determine the position of tangent lines. However, the situation is completely different concerning stretched chords, whose position is always unambiguously fixed. Therefore, the idea appeared that when determining the curvature of the track, not tangential, but corresponding chords should be used. It was assumed that for the considered small track sections the tangents and corresponding chords are parallel to each other, and the points of contact project perpendicularly to the center of a given chord. The moving chord method was first presented in [7]. Figure **2** shows a schematic diagram of curvature determination using the proposed method.



2. Schematic diagram of the moving chord method

The curvature κ_i at a given track point is given by the following formula:

$$\kappa_i = \frac{\Delta\Theta_i}{l_c} \quad (4)$$

where l_c is the length of the chord, and the angle results from the difference of the inclination angles of the chords drawn from point i forward and backward, i.e.

$$\Delta\Theta_i = \Theta_i^+ - \Theta_i^- \quad (5)$$

The application of the discussed procedure requires having the coordinates of a given curve in the Cartesian system (written analytically or in a discrete way), because the values of angles Θ_i^+ and Θ_i^- result from the inclination coefficients of the straight lines describing both chords.

Method verification on a model geometric system

In the work [7], a preliminary verification of the moving chord method was carried out on an elementary geometrical system of tracks, consisting of a circular arc and two symmetrically arranged transition curves (of the same type and length). The universal mathematical notation of such a system was presented in [5]. As part of the verification, four cases were considered - two for the speed of 120 km/h and 100 km/h, with the types of transition curves and the angles of turning of the route diversified. Two types of spirals were used:

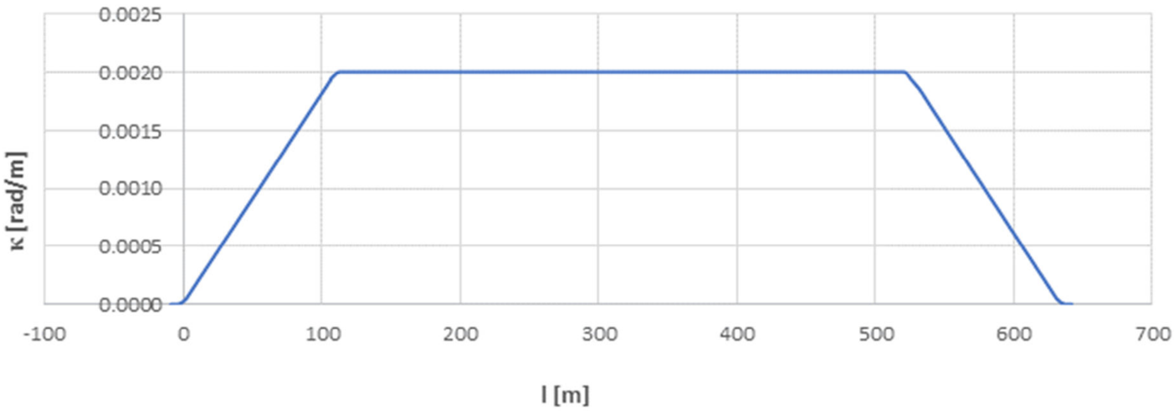
- a commonly used curve in the form of a clothoid,
- a new transition curve proposed in [6].

The procedure consisted of two main stages. First, the coordinates of the successive points of the curve, spaced apart - in a straight line - by the value of l_c (i.e. by the length of the chord) were determined. This was not a problem thanks to the availability of the equations of the individual geometric elements. As part of the verification, $l_c = 5$ m was assumed. In the second stage, the curvature of the track axis was determined using formulas (4) and (5).

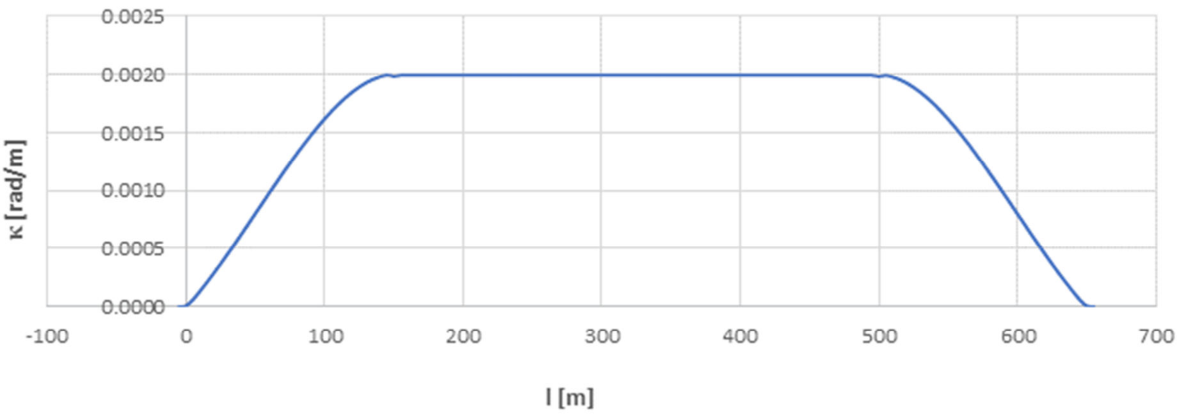
In the second stage, the curvature of the track axis was determined using formulas (4) and (5). The main effort was focused on determining the values of angles Θ_i^+ and Θ_i^- Fig. **3** shows an exemplary graph of curvature $\kappa(l)$ determined by the moving chord method for the selected case: radius of the arc $R = 500$ m, transition curve in the form of a clothoid with length $l_k = 110$ m, angle of turn of the route $\alpha = \pi/3$ rad .

The verification carried out fully confirmed the correctness of the proposed method for determining the curvature of the track axis. In all considered cases, the curvature diagrams showed full compliance with the diagrams constituting the basis for obtaining the corresponding geometric solution. Along the length of the circular arc, the value of the curvature is constant and for the nominal R = 500 m, it gives the value of the occurring radius equal to 499.9979 m, regardless of the type of transition curve used. On the other hand, for nominal R = 800 m, the radius value resulting from the determined curvature is equal to 799.9987 m, for both types of transition curves.

As for the transition curves, a model (i.e. linear) course of curvature was obtained for the clothoid (Fig. **3**) and a smoothed curvature in the final region for the new transition curve (Fig. **4**). In the case of the new curve, there is some slight disturbance locally in the transition from the transition curve to the circular arc. This disturbance can be easily reduced by shortening the chord, but it does not seem to be advisable due to the loss of the possibility of finding the boundary between the transition curve and the circular arc (in the case of the clothoid, this boundary is the breaking point of the straight segments on the curvature diagram).



3. Graphs of the ordinates of curvature $\kappa(l)$ determined by the moving chord method (R = 500 m, clothoid l_k = 110 m, $\alpha = \pi/3$ rad) [7]

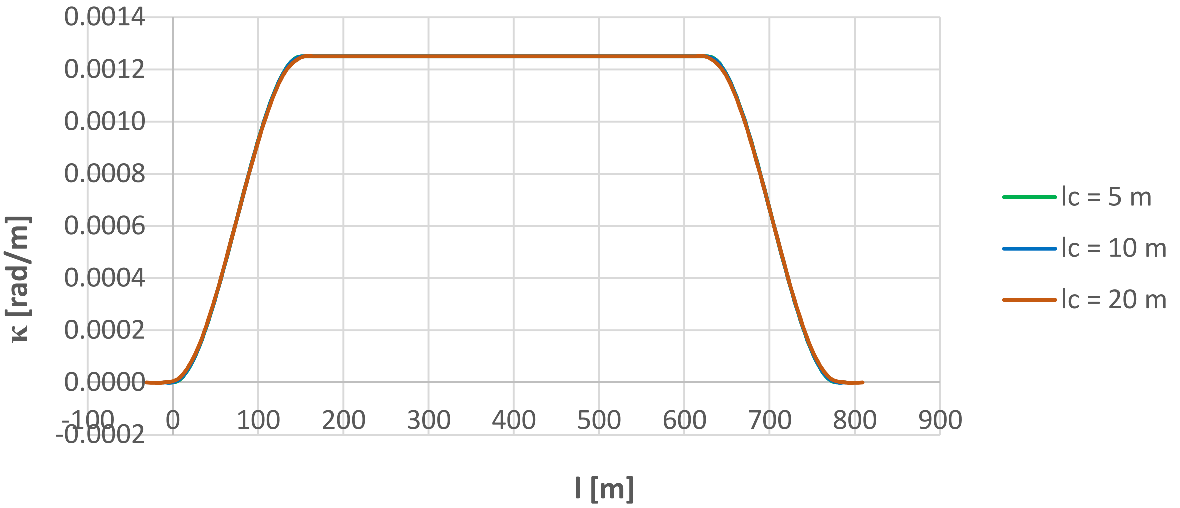


4. Graphs of the ordinates of curvature κ (l) determined by the moving chord method (R = 500 m, new curve l_k = 150 m, $\alpha = \pi/3$ rad) [7]

In the work [8], two important detailed issues were addressed: the influence of the chord length on the curvature values obtained and the possibility of determining the location of boundary points between individual geometric elements. The influence of the length of the movable chord was considered - similarly to the work [7] - for the elementary geometrical system determined according to the principles of the analytical design method [5]. The path turn angle $\alpha = \pi/4$ rad and the speed of the trains were assumed V = 120 km/h (which results in a circular arc radius of R = 800 m with a cant of h = 85 mm). The analyzed variants resulted from the type of transition curves used. The lengths of these curves were varied, resulting from the need to maintain the permissible values of the relevant kinematic parameters. Accepted following types curves transitional:

- a curve in the form of a clothoid with a length of 105 m,
- Bloss curve with a length of 150 m,
- a new transition curve [6] with a length of 135 m.

In the considered cases, the curvature values were determined for the assumed lengths of the movable chord l_c = 5 m, 10 m and 20 m. It was found that in each of the variants, the graphs of the curvature ordinates coincide with each other. Figure **5** shows exemplary graphs obtained for the Bloss curve.



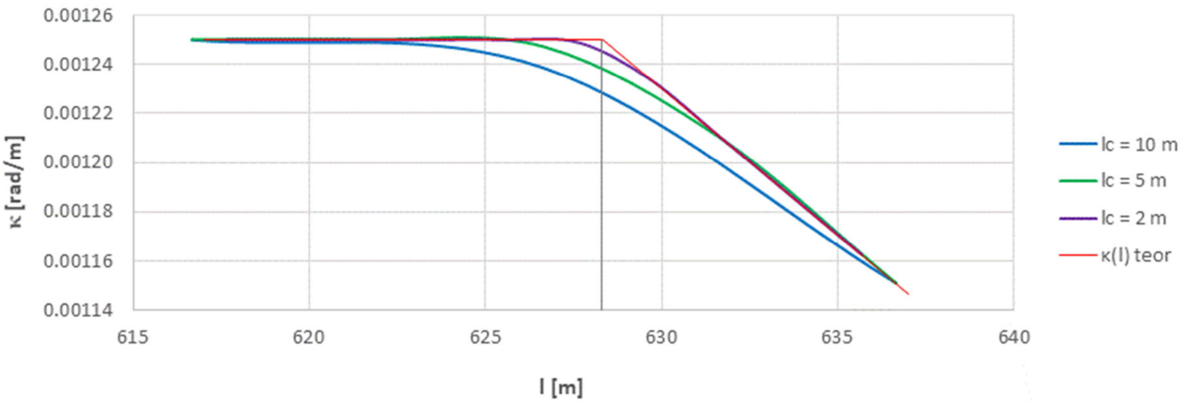
5. Curvature ordinate plots $\kappa(l)$ for assumed movable chord lengths $l_c = 5$ m, 10 m and 20 m ($\alpha = \pi/4$ rad, $R = 800$ m, Bloss curve $l_k = 150$ m) [8]

This means that in the range $l_c = 5 \div 20$ m, the length of the chord does not play a significant role in determining the curvature and does not limit the use of the discussed method. At the same time, the precision of determining the nature of the curvature and the consistency with the theoretical course on the transition curves are noteworthy. Both transition curves in the form of a clothoid have a linear curvature, Bloss curves - curvature in the form of the letter S, and curves [6] - curvature smoothed in the end regions. Relatively small discrepancies in the ordinates of curvature occur only in the regions of transition from straight sections to transition curves and from transition curves to a circular arc

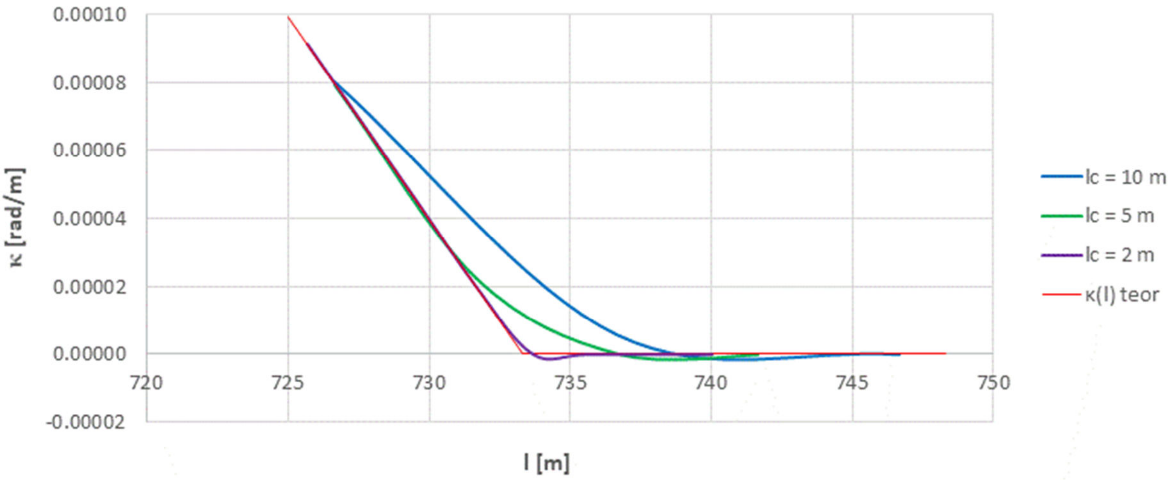
Since - as it has been shown - the determination of the curvature along the track length is not a special problem, it remains to be clarified the issue of the location of the connection points of geometric elements - straight sections with transition curves and transition curves with circular arcs. This may prove particularly important from a practical point of view.

The analysis showed that in the case of model (theoretical) geometrical systems, the long chord ($l_c = 20$ m) cannot be used to achieve the intended goal. A chord of length $l_c = 5$ m is suitable only when using the Bloss curve . For the *clothoid and the* **new** curve **, an even shorter** chord should be used , i.e. $l_c = 2$ m. curve to a straight line. Figures **6** and **7** show the corresponding graphs of curvature $\kappa(l)$ in the region of transition from a circular arc to a transition curve in the form of a clothoid and in the region of transition from this curve to a straight line. The theoretical curvature is marked in red.

The sought value of the abscissa of the connection point in Figure **6** is $l = 628.3185$ m, and in Figure **7** - $l = 733.3185$ m. As can be seen, a chord of length $l_c = 2$ m is close to obtaining such values. The analysis shows that in the moving chord method, it is possible to determine the location of boundary points between individual geometric elements, while the required length of the chord must be adapted to the type of transition curve.



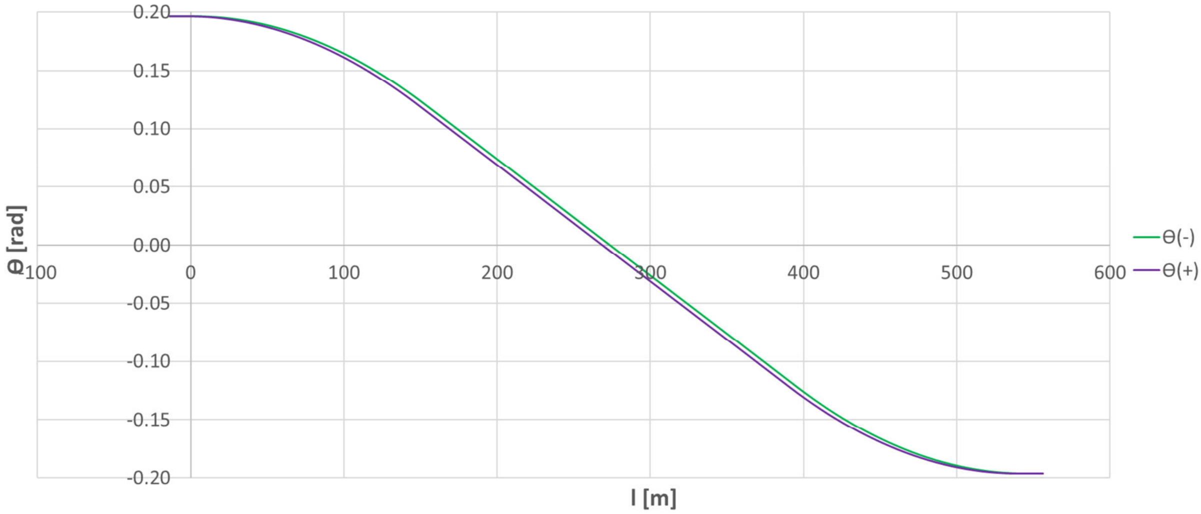
6. Graphs of the ordinates of curvature *κ* (*l*) in the region of the transition from a circular arc to a transition curve in the form of a clothoid for the assumed lengths of the moving chord $l_c = 2$ m, 5 m and 10 m ($\alpha = \pi/4$ rad, $R = 800$ m, $l_k = 105$ m) [8]



7. Graphs of the ordinates of the curvature *κ* (*l*) in the region of transition from the transition curve in the form of a clothoid to a straight line for the assumed lengths of the moving chord $l_c = 2$ m, 5 m and 10 m ($\alpha = \pi/4$ rad, $R = 800$ m, $l_k = 105$ m) [8]

In the paper [9], the universality of the discussed method was indicated - it can be used concerning both the horizontal and vertical planes. The focus was on the computational basis for the angles of inclination of the moving chord. The analysis of inclination angles in the horizontal plane was carried out in the *x, y* rectangular coordinate system, on the elementary geometrical system of tracks, consisting of a circular arc with a radius of 1000 m and two symmetrically arranged transition curves in the form of a clothoid with a length of 150 m. Assuming the cant value h_O = 105 mm allows obtaining the speed of trains V = 140 km/h .Three special cases were considered, differing in the angle of turn: $\alpha = \pi/8$ rad, $\pi/4$ rad, and $\pi/2$ rad. As part of the analysis, the chord length l_c = 5 m was assumed

The curvature values result from the difference in the inclination angles of the chords derived from point *i* – forward Θ_i^+ and backward Θ_i^-. Figure **8** shows graphs of both of these angles along the length of the considered geometrical system for the turn angle of the route $\alpha = \pi/8$ rad. It is clearly visible that the length of the circular arc (in the middle part) maintains a fixed distance between the two graphs (i.e. they are parallel to each other). Dividing the constant value of the difference of the angles by the length of the chord, one obtains the constant value of the curvature. Along the length of the transition curves, the angle difference decreases to zero on straight sections; in the case under consideration, this leads to a linear curvature.

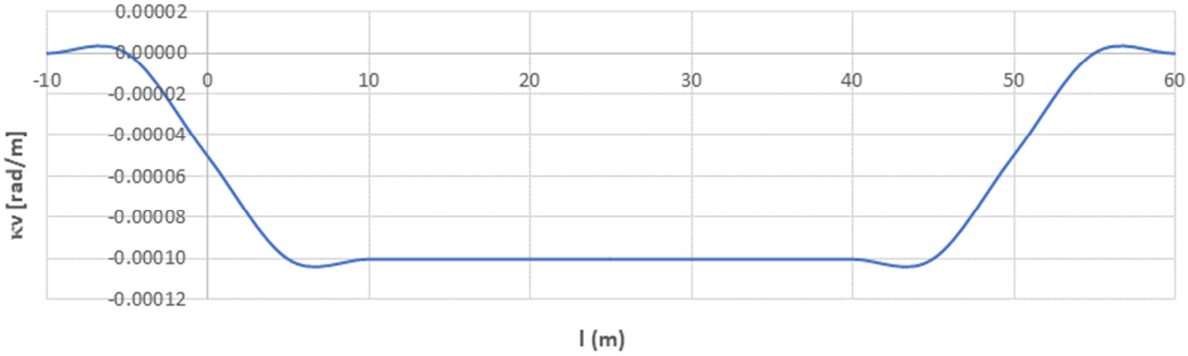


8. Graphs of the angles of inclination of the chords derived from point Θ_i^+ *i* – forward and backward Θ_i^- for the selected system (radius of the circular arc R = 1000 m, transition curves in the form of a clothoid with a length of l_k = 150 m, angle of turn of the route $\alpha = \pi/8$ rad) [9]

As stated, the situation is analogous in the case of angles $\alpha = \pi/4$ rad and $\pi/2$ rad – the values of the angles of inclination of the moving chord are the greater the greater α *is* . In the considered cases, the order of magnitude of the angles and for individual α differs significantly (ranges from 0.1 to 0.7 rad), but the differences of the corresponding angles are the same. On a circular arc with a radius of 1000 m, they are 0.005 rad, with a curvature of 0.001 rad/m. Thus, for a circular arc in the horizontal plane, the values of the angles of inclination of the moving chord depend on the radius of the arc and the angle of turn of the route, while the difference in the angles of inclination depends only on the radius of the arc.

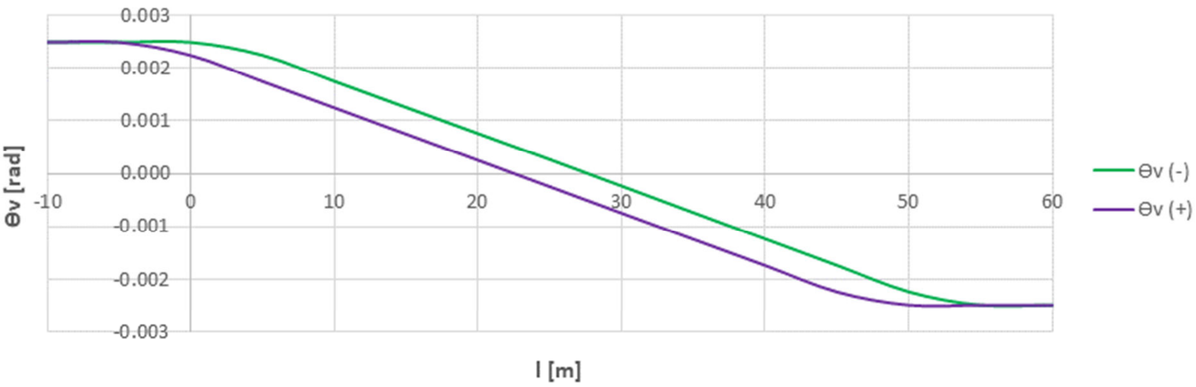
From the formal point of view, nothing stands in the way of the moving chord method being also used to determine the vertical curvature. The analysis of the angles of inclination of the movable chord in the vertical plane was carried out on a geometric system consisting of two symmetrically arranged sections with a uniform inclination equal to 2.5% o and a circular arc with a radius of 10,000 m. With the coordinates of individual points determining the ends of the movable chord, the values for each point *i* were determined angles of inclination of the two adjacent chords. Figure **9** shows the obtained graph of the ordinates of the vertical curvature $\kappa_v(l)$ determined by the moving chord method with a length of l_c = 5 m.

Similarly to the horizontal plane, in the central part of Figure 9 (i.e. on the circular arc) there is full compliance with the model solution - the curvature of the circular arc is a constant value equal to 0.0001 rad/m. In the extreme regions, however, there is a variability of the curvature, which is undoubtedly related to the length of the adopted chord. Therefore, appropriate calculations were also carried out for a 2 m chord, resulting in a radical shortening of the transition zones.



9. Graph of the vertical curvature $\kappa_v(l)$ on the length of a geometric system consisting of two segments with a uniform slope equal to 2.5‰ and a circular arc with a radius of 10,000 m (chord length l_c = 5 m) [9]

The values of the vertical curvature result from the difference in the angles of inclination of the chords Θ_{vi}^{+} and Θ_{vi}^{-}, derived from the point *i*. Fig. **10** shows graphs of these angles along the length of the geometric system for the length of the chord l_c =5 m



10. Graphs of the angles of inclination of the chords in the vertical plane derived from point *i* – forward Θ_{vi}^{+} and backward Θ_{vi}^{-} for l_c = 5 m (two symmetrically arranged sections with a uniform slope equal to 2.5‰, radius of the circular arc *R* = 10,000 m) [9]

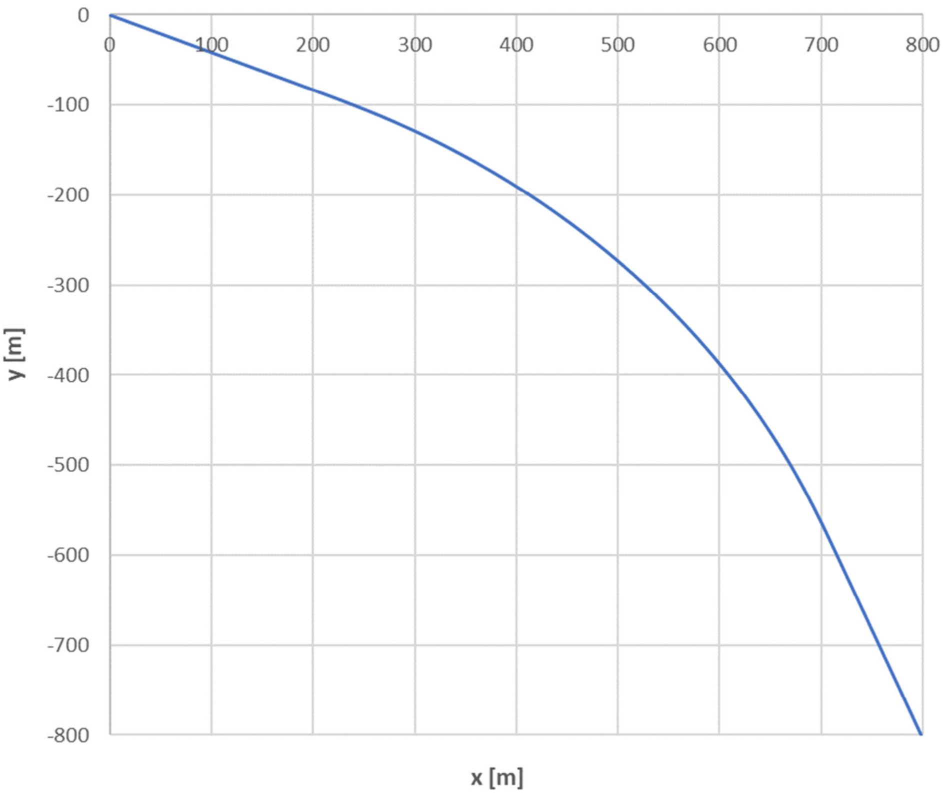
Apparently, as in the case of the horizontal plane, a fixed distance between the two graphs is maintained along the length of the circular arc (it drops to zero on the extreme sections). Dividing the constant value of the angle difference of 0.0005 rad by the length of the chord gives the constant value of curvature of the vertical arc κ_v = 0,0001 rad/m. The value of the angle difference in the horizontal plane for the same chord length was 0.005 rad on a circular arc.

To sum up, it should be stated that, similarly to the horizontal plane, the radius of the vertical arc is the only factor determining the value of the difference in the inclination angles of the moving chord. However, it should be taken into account that in the case of a circular arc in the vertical plane, the values of the angles of inclination of the moving chord are much smaller than in the horizontal plane, which is related to the range of arc radii used

Determination of application possibilities

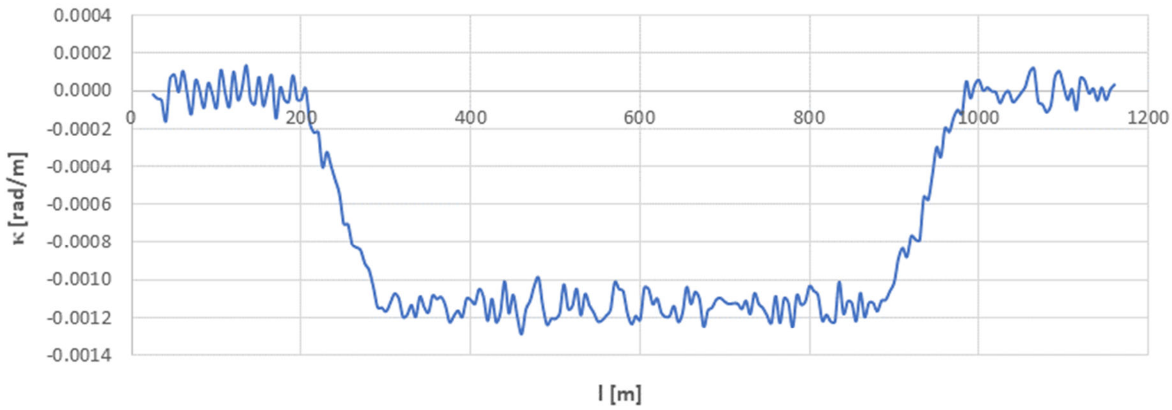
The verification of the movable chord method on a model geometric system was essentially of a cognitive nature. It confirmed the substantive correctness of the discussed method, but the key issue remains its use to determine the horizontal curvature of the axis of the operated railway track, based on Cartesian coordinates obtained by direct measurements. In these measurements, we consider a geometric system with unknown characteristics and we have no possibility to operate with mathematical notation. Therefore, the basic problem here will be to determine the coordinates of the ends of both chords by interpolation carried out in the appropriate intervals. After determining the angles Θ_i^+ and Θ_i^-, the curvature values - as before - are determined using formulas (4) and (5). The entire course of the proceedings is sequential and consists in the use of appropriate calculation formulas.

The practical use of the moving chord method was presented on the example of curvature estimation for a test section of a railway line about 1200 m long. The Cartesian coordinates of this section were determined at intervals of about 5 m, and the maximum error of this operation was $\pm$25 mm.



11. The route on the considered test section

Figure **11** shows the course of the considered route in the local coordinate system. As you can see, there are two straight sections and a circular arc (probably with transition curves). Other than that statement, Figure **11** doesn't tell us much. In order to be able to fully identify a given geometrical system, the curvature of the track axis must be determined (Fig. **12**). The length of the moving chord l_c = 20 m was assumed.



12. Curvature plot κ(l) along the length of the geometric system shown in Figure **11** obtained using the moving chord method ($l_c = 20$ m)

The horizontal curvature diagram obtained for the measured railway track clearly differs from the diagrams for model systems presented in [7-9]. It has an oscillating character, which results from the track deformations and the measurement error. However, as it turns out, from the point of view of practical use of this graph (to identify the geometrical system), it does not matter. As with model systems, the graph $\kappa(l)$ in Figure **12** consists of elements of two kinds:

- segments oscillating around a horizontal course, which describe a curvature of a fixed value (zero on straight track sections and non-zero on circular curves), and
- segments oscillating around a linear course (i.e. straight lines inclined to the *l axis*), which describe the variable curvature present in the transition curves.

On the basis of this graph, you can determine the value of the radius of the circular arc and the length of the spirals, as well as the location of characteristic points (lying at the connections of straight segments with spirals and spirals with a circular arc). It is assumed that the curvature of the track on straight sections is equal to zero, and the disturbances on the curvature diagram appearing there are the result of the existing deformations and the measurement error. From the selected range of curvature values unquestionably belonging to a circular arc, the arithmetic mean is determined $\overline{\kappa_{ŁK}}$; its inverse determines the value of the radius:

$$R \cong \frac{1}{\overline{\kappa_{ŁK}}} \tag{6}$$

The value obtained using the formula (6) should be properly rounded (to full meters) and then used in the further calculation procedure. In the considered case, the mean value $\overline{\kappa_{ŁK}} = -0.001136691$ rad/m was obtained for the first circular arc , which corresponds to the value of the radius $R = 879.74625$ m. Thus, a radius of 880 m can be assumed for further calculations.

There are spirals on both sides of the circular arc. For example, at the end of the line *P1* on the left in Figure **12** there is the beginning of the transition curve *KP1* connecting this line with the circular arc. In turn, at the beginning of the straight line *P2 on the right side of this curve, the beginning of the transition curve KP2* is located . The ends of both curves, i.e. the points *KKP1* and *KKP2* , determine - respectively - the beginning and the end of the circular arc.

In order to determine the linear coordinates of the mentioned characteristic points, it is necessary to determine the coefficients of the least squares lines describing the regions of the *k* (*l*) graph with variable curvature values. Lines of least squares determine the linear coordinates of their intersections with curvature graphs on straight track segments (where k = 0 curvature) and circular arc segments (where *κ curvature* $= \overline{\kappa_{ŁK}}$).).

KP1 results directly from the values of the determined coordinates l_{PKP1} and l_{KKP1} .

$$l_{KP1} = l_{KKP1} - l_{PKP1} \tag{7}$$

the length of the transition curve KP2 can be determined from the values of the determined coordinates l_{PKP2} and l_{KKP2}.

$$l_{KP2} = l_{PKP2} - l_{KKP2} \tag{8}$$

The numerical data resulting from the formulas (6) ÷ (8) and the appropriate Cartesian coordinates of the determined characteristic points fully identify the measured geometrical system. The new method of curvature determination fully proves its usefulness here. Occurring oscillations on the curvature diagram obtained do not interfere with the estimation of the basic geometrical parameters of the measured system.

The issue of using the direction angle of the route

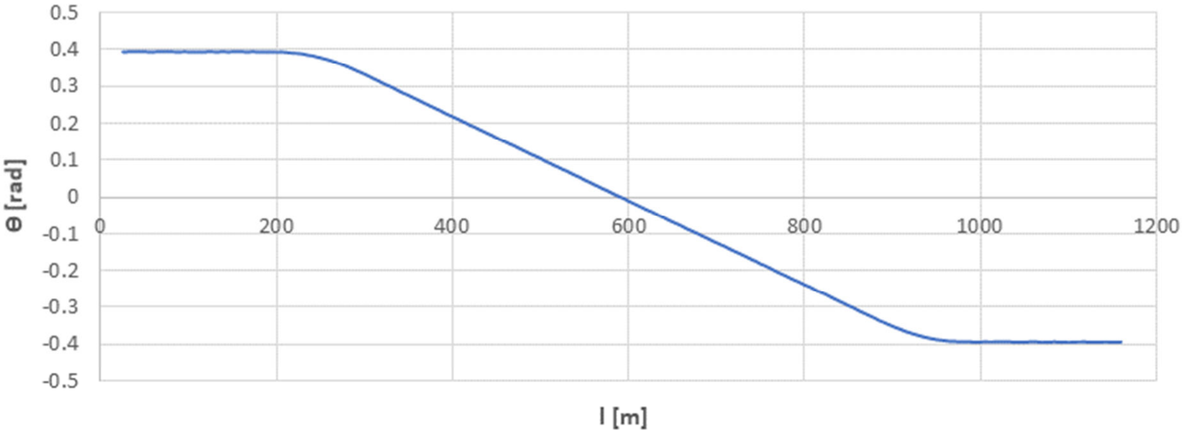
Determined during the procedure of determining the curvature at the measurement point *and* the values of the inclination angles of both virtual chords derived from this point to the abscissa of the appropriate rectangular coordinate system can be simultaneously interpreted as direction angles of the route. They do not apply to a given measurement point, but to points distant from the point *and* forwards and backwards by the value corresponding to half the length of the moving chord. In order to be able to plot the dependence $\Theta(l)$ - separately for each chord, linear coordinates of these points would have to be determined, which, however, would require an additional calculation procedure.

However, as it turns out, this problem can be solved in a much simpler way, using the angles of inclination of both chords. These angles make it possible to directly determine the directional angle of the route at point i as the value of their arithmetic mean.

$$\Theta_i = \frac{\Theta_i^+ + \Theta_i^-}{2} \tag{9}$$

It can be assumed that the angle Θ_i is also the angle of inclination of the tangent to the geometric system at a given measurement point. If its analytical notation in the form of the function $\Theta(l)$ was known, the curvature could be determined from the formula (2). However, since on the basis of the measurement data such a record cannot be precisely determined for the non-linear segments appearing in the $\Theta(l)$ graph, it is necessary to limit the inference to the regions where the empirical $\Theta(l)$ graph is linear. Horizontal segments (where the value of the derivative is zero) correspond to straight route segments, while segments with a fixed inclination - circular arcs (where the derivative is a constant value and from it it is possible to determine the radius value as the inverse of the existing curvature).

Using the same calculation data as for determining the curvature (i.e. the values of angles Θ_i^+ and Θ_i^-) , a plot of the directional angle of the route shown in Figure **11** was made , determined by formula (10). This chart has been left presented on drawing **13** .



13. Diagram of the directional angle along the length of the geometric system shown in Figure **11**, obtained using the moving chord method ($l_c = 20$ m)

The directional angle plot in Figure **13** confirms the observations resulting from the curvature plot in Fig. **12** concerning the characteristics of the considered geometric system. It gives a general orientation regarding the location of straight sections and circular arcs (where the graph is linear). The undefined (i.e. non-linear) nature of the graph on both transition curves makes it impossible to determine the location of the boundary points between the individual geometric elements, which is a key issue in this case. Such a situation (and the related lack of knowledge of the length of the transition curves) makes reliable identification of the geometric system ineffective. When using the information resulting from the curvature diagram (Fig. **12**), the situation is completely different, so there is no need to promote the use of an approach that is not fully valuable from the point of view of engineering practice.

It should also be mentioned that recently - as part of scientific work - an attempt was made to use the directional angle diagram to determine the curvature of the track axis and, consequently, to identify the geometrical arrangement. Curvature estimation on the transition curves is performed here in an approximate way - either by numerical differentiation of the non-linear fragment of the graph $\Theta(l)$ or by approximating the points of this graph using a quadratic function (in order to obtain the predicted linear curvature). Of course, it is not possible to precisely determine the location of the boundary points, so the determined geometric parameters are also approximate. The geometric system prepared on the basis of these parameters will therefore not coincide with the measured system, and yet the compliance of the model system with the measured one determines the correctness of the applied method of identifying the track axis.

Summary

Determination of the basic geometrical parameters of the railway route in the horizontal plane (i.e. determining the location and length of straight sections, the location of circular arcs along with determining their radius and length, and the location of transition curves along with determining their type and length) is most often done based on measured arrows from the chord stretched along the track. The reference line here is the directions of the chord settings, which are constantly changing. The use of horizontal arrows stems from the lack of a direct method for determining the curvature of the track axis.

From the definition of curvature, it is necessary to operate with the angles of inclination of the tangent to the geometric system. In a real railway track reconstructed by measurements, it is very difficult to determine the position of tangent lines. Therefore, the idea appeared that when determining the curvature of the track, not tangential, but corresponding chords should be used. In this way, the idea of a new method of determining the curvature of the track axis was created, based on the use of the difference in the angles of the chord of a fixed length at a given point

This paper presents a summary of the current activities related to the verification of the proposed method of determining the curvature (the so-called moving chord method) on a clearly defined model geometric layout of tracks, determined following the principles of the analytical design method [5]. The curvature diagrams obtained for several considered cases were shown, in which the speed of trains, the types of applied transition curves, and the angles of turning of the route were differentiated. The issue of the location of the connection points of geometric elements - straight sections with transition curves and transition curves with circular arcs has been clarified, which may be particularly important from a practical point of view. The focus was also on the computational basis of the discussed method, regarding the angles of inclination of the moving chord. Its universality was also pointed out - the possibility of using it in relation to both the horizontal and vertical planes. The analysis carried out showed full compliance of the obtained curvature diagrams with the diagrams constituting the basis for obtaining the corresponding geometric solution. This concerned both circular arc segments and transition curve regions.

Then, the key issue was taken up - the use of the moving chord method to determine the horizontal curvature of the axis of the operated railway track, based on Cartesian coordinates obtained by direct measurements. It was found that the curvature diagrams prepared based on measurement data differ from the diagrams for model systems - they have a less regular, oscillatory character, which results from the track deformations and the measurement error. However, this does not interfere with the estimation of the basic geometrical parameters of the measured system. Based on the curvature diagram, you can determine the value of the radius of the circular arc and the length of the spirals, as well as the location of characteristic points. The new method of curvature determination fully proves its usefulness here.

The general orientation regarding the location of the existing straight sections and circular arcs is provided by the route direction angle diagram, which can be easily determined as part of the curvature determination procedure. However, the undefined (i.e. non-linear) nature of this graph on the spirals makes it impossible to determine the location of the boundary points between the individual geometric elements (and thus also the length of the spirals). This situation makes the reliable identification of the geometric system based on the directional angle ineffective. When using the information resulting from the curvature diagram, the situation is completely different, so there is no need to promote the use of an approach that is not fully valuable from the point of view of engineering practice.

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