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Dynamic systems for the analysis of the behavior of geosynthetics in railway engineering structures

Abstract: While interacting with the elements of the engineering structures, geosynthetics can be treated as elastic membranes or shells placed on different types of foundation. Modeling of the real system takes into account the most important properties of the system and its elements. We will develop a physical and mathematical model in a form of generalized dynamic system. The mathematical description will use different operators leading to a continuous distributed system. The modeling will be further modified by development of discrete dynamic systems, which is enabled by the theory of generalized dynamic systems. This approach allows for the analysis of the problem with continuous and discrete signals. The results will show the response of the analyzed systems with analytic, numerical or hybrid methods.

Keywords: Geosynthetics; Dynamic systems; Non-classical operational calculus

Introduction

In the description (modelling) of many physical and mechanical systems there are used continuous and discrete dynamic systems with concentrated and distributed parameters, time variant and time invariant. Wide capabilities of this theory, primarily general theory of dynamic systems allow to apply them in the behavior analysis of geosynthetics in rail engineering constructions by using special differential or difference operations as well as suitable shape of endomorphisms.

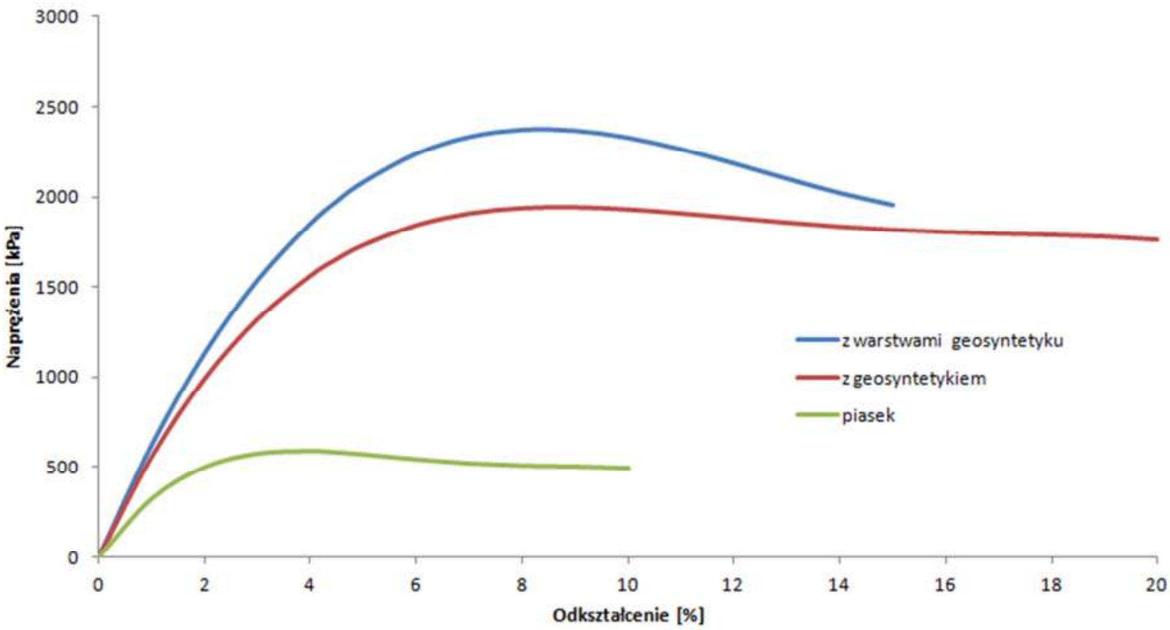
All those manipulation, what is important, have mathematical ground and thus their usage is authorized, about which we will see later.

Geosynthetics, in other words geofabrics, geofibers, geonets, geomembranes, geocoverings etc. are widely used in architecture, including construction, modernization and renovation of railroad tracks according to the guidelines included, among others, in [17]. Well-constructed railroad track is not a subject to random and uncontrolled deformations. Increase of bearing capacity of individual ground layers creating ground structure has particular importance in construction of railroad tracks.

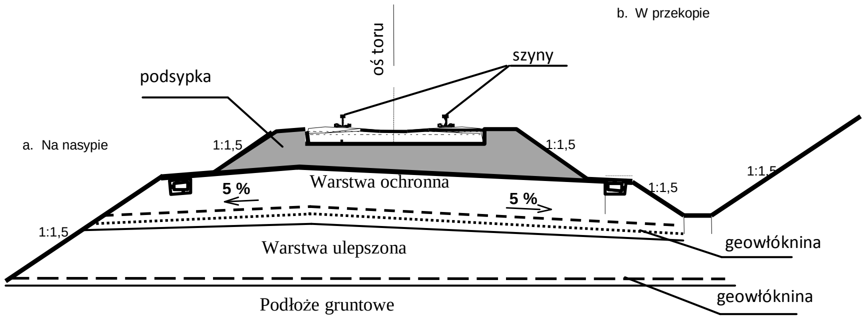
Common feature of geotextiles, geofabrics and geomembranes is lack of bending stiffness. Their mechanical properties depend on the way they are made, used materials and their anchoring methods. Often geosynthetics used in constructions have multilayered placement (fig. **1**).

Such a multilayered use of geosynthetics is beneficial. This is confirmed by tests carried out in a triaxial shear apparatus for soil samples (fig. **2**). This multilayering in cross section of railroad track is shown in fig. **3.** From aforementioned it follows, that every next layer has different loading and cooperation. It is schematically illustrated on fig. **4.**

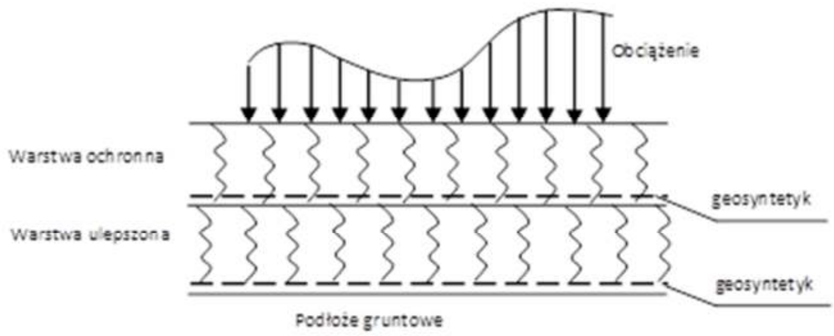
1. Multilayered positioning of geosynthetics on earthwork



2. Characteristics of tests carried out in a triaxial shear apparatus for soil samples of different geosynthetics content based on [5] with $\sigma_3 = 100\ kPa$



3. Example schematic of geosynthetics placements in railroad track cross section. [11]



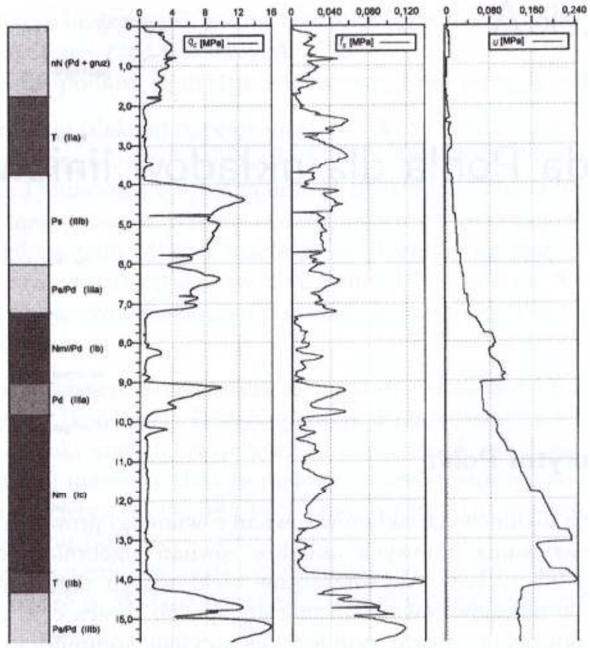
4. Example schematic of load (substitute loads) layers of railroad track [11]

Analog characteristics of analyzed problem

In many cases of analysis of geosynthetics cooperation with the ground and elements of engineering constructions (e.g. subgrade) we treat selected geosynthetics as elastic membranes or coatings based on different types of substrate, e.g. Winkler type, Euler, Pasternak, Kerr, as modelling the real system means idealizing it in consideration of those environment features which appear to be most important from the point of view of analyzed problem.

Built physical system, and later mathematical, is usually shown in form of detailed general cases of dynamic systems [13]. In this detailed description general theory of dynamic systems [13] allows the use of Laplace operator Δ, and later with it's help d'Alembert operator $\square$. Similarly, the plate on a subsoil (elastic foundation) is described using bilaplasian [16].

Similarly (in terms of differential problems) are modeled plate systems, foundations [8], slab – pile foundations [6, 15, 18, 21] by choosing the right system parameters, taking into account the parameters of the subsoil [19]. On plates, on slab – pile foundations there are placed railway lines, tram lines, pipelines and runways. In those cases geosynthetics are also used. In [4] authors use in the modeling elastic foundation to evaluate impact of uneven subsidence of the bottom of the tank on the distribution of internal forces in its bottom. It is important for determining failure rate of big tanks [1]. Such concepts are used in the analysis of the interaction in the railway vehicle - track - subgrade - subsoil system including their experimental verification [14], also by early identification of the subsoil, e.g. with the use of probes [7] and determining its parameters using CPT method without measuring water pressure in soil pores or the CPTU method with measuring water pressure in soil pores. Results of probing can be represented by appropriate graphs (fig. **5**). Graphs record resistance on the taper probe q_c, friction on the friction sleeve f_s and pressure of water in soil pores u.



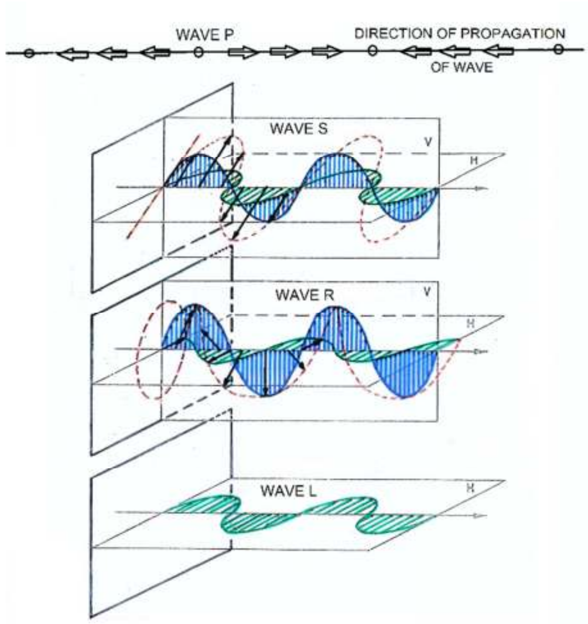
5. Example result of CPTU research in chosen point (node) of probing [9]

In the case of railway lines, the behavior of the structure is influenced by vibrations that are caused by train movements and depend, among others, on their speed. After exceeding the so-called critical speeds track and subgrade vibrations increase significantly and permanent track deformations increase rapidly [3]. This is due to the propagating vibrations (waves), which are a kind of perturbations in the velocity field, moving in the medium taking various forms and speeds. Elastic waves are decisive here. These are mechanical waves that propagate in the elastic medium that is the subgrade and then pass into the environment as a result of the forces associated with the deformation of the volume and form of the elements of the entire medium. External objects causing these deformations are passing rail vehicles, which are wave sources, and their generation is also affected by the geometry and condition of the track.

Propagation of elastic waves in the subgrade is basically based on the excitation of medium particles more and more distant from the source of the waves. Elastic waves (fig. **6**) can be divided into volumetric waves propagating in soil and air (noise) and surface waves propagating along surfaces separating media with different properties, including waves propagating on the ground surface.

Longitudinal volume waves P reach the vibration recorder, the receiver, first. These waves cause vibration deviation in a direction parallel to the direction of wave propagation. They compress and stretch the medium and propagate at the speed c_L. Transverse volume waves S reach from the source to the recorder of vibrations after the wave P and have a speed c_S. The velocity of the P and S waves depends on the size of the elastic parameters of the medium and with the change of these parameters these velocities may differ significantly, and especially these changes may be associated with, for example, an increase in depth in the soil [12].

Surface waves: Rayleigh - R and Love - L waves have long periods and varying amplitudes, with the amplitude of their vibrations decreasing exponentially as the depth increases. Surface Rayleigh R waves, whose speed depends on their frequency, propagate horizontally and cause both vertical and horizontal, but not often transverse movements of the ground surface (fig. **6**). The vertical and horizontal components are opposite in phase, so that the motion of the particles is elliptical - takes place on an ellipse oriented vertically and which is perpendicular to the direction of the wave. Under the influence of these vibrations, the soil grains move along tracks similar to ellipses, while the soil loosens strongly. Love L waves also propagate horizontally, causing horizontal, lateral movements of the particles (fig. **6**).



6. Waves P, S, R, L [12]

From this it follows that in many cases it is necessary to strengthen the subgrade, including using geosynthetics. Comparison of the effectiveness of some ground reinforcements for high-speed lines is presented, for example, in [20].

Generalized dynamical systems in the non-classical operator calculus and their application

Generalized dynamical systems [13] are conceptually related to the non-classical operator calculus, which is based on three linear operations S, s, T and two linear spaces L^1, L^0. We assume that $L^1 \subset L^0$, $S:L^1 \to L^0$,, $T:L^0 \to L^1$, $s:L^1 \to \mathrm{Ker}S$ and additionally that ST = id, TS = id-s.

Uogólnione układy dynamiczne [13] pojęciowo związane są z nieklasycznym rachunkiem operatorów, który bazuje na trzech liniowych operacjach S, s, T i dwóch przestrzeniach liniowych L^1, L^0,. Zakładamy, że $L^1 \subset L^0$, $S:L^1 \to L^0$,, $T:L^0 \to L^1$, $s:L^1 \to \mathrm{Ker}S$ oraz dodatkowo, że ST = id, TS = id-s. Properties of S, T, s operations and their various representations can be found in [2, 10, 13]. Importantly: these operations can be defined in a continuous or discrete domain and then they lead to a description using differential or difference equations or their systems. Therefore, they can be used to describe and analyze generalized dynamical systems [13]. This approach leads to the generalized Taylor formula of the character

$$x = sx + TsSx + T^2 sS^2 x + \ldots + T^{n-1} sS^{n-1} x + T^n sS^n x, \qquad (1)$$

where $x \in L^n \left\{ x \in L^{n-1} : Sx \in L^{n-1} \right\}$.

Attention! In the last formula *x* does not mean the coordinate of the point, but an element of any linear space L^n defined earlier. This shows the generality of the formula (1).

It turns out that for the analysis of our problem, Laplace operator should be adopted as the operation S.

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

or d'Alembert operator

$$\square = \Delta - \frac{1}{a^2}\frac{\partial^2}{\partial t^2} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{a^2}\frac{\partial^2}{\partial t^2}$$

and choose for them operations T and s.

Attention! In case of n=2 Laplace operator is reduced to the form of $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$**,** however dla n=1 we only get $\frac{\partial^2}{\partial x^2}$ and this changes the differential equation of the model [11].

In case of operation □ (n=3) operations T and s are defined by formulas

$$T\left\{ f\left(P,t\right) \right\} = \left\{ -\frac{1}{4\pi} \iiint_{\Omega} \frac{Af\left(P_0,t\right)}{d\left(P,P_0\right)} d\Omega \right\}, \tag{2}$$

$$s\left\{ u\left(P,t\right) \right\} = \left\{ \frac{1}{4\pi} \iint_{\sigma} \left[\frac{1}{d\left(P,P_0\right)} A\left(\frac{\partial u}{\partial n}\right) - Au \frac{\partial}{\partial n}\left(\frac{1}{d\left(P,P_0\right)} \right) + \frac{1}{ad\left(P,P_0\right)} A \cdot \left(\frac{\partial u}{\partial t}\right) \frac{\partial d\left(P,P_0\right)}{\partial n} \right] d\sigma \right\} \tag{3}$$

where: $L^0 = C^1\left(\Omega \times < 0,\infty)\right)$, $L^1 = C^3\left(\Omega \times < 0,\infty)\right)$,

operation A is substitution for *t* values $t - \frac{d\left(P,P_0\right)}{a}$ and $a \neq 0$ is the constant characterizing the replacement system

$d\left(P,P_0\right) = \sqrt{\left(x-x_0\right)^2 + \left(y-y_0\right)^2 + \left(z-z_0\right)^2}$, $\frac{\partial}{\partial n}$ is the normal derivative, σ is the boundary of the set Ω.

In each of the situations described in mathematical modeling there is a partial differential equation □*u=f(x, y, z, t)*. In our case, it describes a replacement system in which the constant *a* characterizes the substrate (foundation) with geosynthetics, respectively, while *f(x, y, z, t)* characterizes the continuous equivalent load. These values have to be determined empirically [7, 14, 17], including in the laboratory using a three-axis apparatus or approximated by the parameters of the system components (Figures **3** and **4**), which in the model can be treated as connected in series or in parallel and can create a replacement dynamical system [13].

Of course Of course *x, y, z* are the coordinates of the point *P* ϵ Ω at the moment *t*, and *u* is a function of the appropriate class in the set Ω×<0,∞) and means deflection (displacement, change in the position of points P). In our model, it is enough that Ω is a prism, or in particular, a cuboid with appropriate dimensions. Using formulas (1), (2), (3) we can write

$$u\left(x,y,z,t\right) = u\left(P,t\right) =$$

$$= \left\{ \frac{1}{4\pi} \iint_{\sigma} \left[\frac{1}{d\left(P,P_0\right)} A\left(\frac{\partial u}{\partial n}\right) - Au \frac{\partial}{\partial n}\left(\frac{1}{d\left(P,P_0\right)} \right) + \frac{1}{ad\left(P,P_0\right)} \cdot A\left(\frac{\partial u}{\partial t}\right) \frac{\partial d\left(P,P_0\right)}{\partial n} \right] d\sigma \right\} + \tag{4}$$

$$+ \left\{ -\frac{1}{4\pi} \iiint_{\Omega} \frac{Af\left(P_0,t\right)}{d\left(P_0,t\right)} d\Omega \right\}$$

i.e. determine the values of the function *u* at the *P* point inside Ω area at the moment *t*.

From formula (4) it follows that if u_1 is the response of the free system $u = 0$, i.e. it is a spherical wave, while u_2 is the response of the non-free system with forcing (load) *f(x,y,z,t),* then the answer of the *u* system can be represented in the form $u=u_1+u_2$.
If the equivalent load $f \in Ker^n$, then according to formula (1) we can write $u(P,t) = su + Tsf + T^2 s^2 f + \ldots + T^{n-1} s^{n-1} f$,
Where T and s are defined by formulas (2) and (3).

As shown in [11], the change in location of points of the analyzed substrate, including points from geosynthetics, can be estimated by spherical waves (spherical functions).
Attention! The dynamical systems used can also be analyzed in the results space [13], which is richer in elements than the output space.

The properties of solutions (4) [11] entitle to use the presented mathematical descriptions to model changes in the location of points of the analyzed area Ω, in particular points of interest *(x, y, g (x, y))* of geosynthetics, where $z = g(x, y)$ is the equation of the geosynthetic surface, and *(x, y)* belong to the set $D \subset R^2$, which is the Ω projection on the XOY plane of the coordinate system and points lying in layers sufficiently close to the surface of the geosynthetic.

In order to identify the model, it is necessary to conduct experimental research using active, passive or mixed experiment and validate the model.

Discretization of the problem and discrete dynamic system

Let

$$x = i\Delta x; y = j\Delta y; z = k\Delta z; t = l\Delta t$$

where $\Delta x, \Delta y, \Delta z, \Delta t$ are variable increments, respectively x, y, z, t
and

$$u_{i,j,k,l} = u(i\Delta x, j\Delta y, k\Delta z, l\Delta t), \; f_{i,j,k,l} = u(i\Delta x, j\Delta y, k\Delta z, l\Delta t)$$

Replacing partial derivatives with their differential quotients and introducing in the space of four indicator discrete signals operations

$$S_1 u_{i,j,k,l} = u_{i+1,j,k,l}$$

$$S_2 u_{i,j,k,l} = u_{i,j+1,k,l}$$

$$S_3 u_{i,j,k,l} = u_{i,j,k+1,l}$$

$$S_4 u_{i,j,k,l} = u_{i,j,k,l+1}$$

or

$$S_1 u_{i,j,k,l} = u_{i+1,j,k,l} - u_{i,j,k,l}$$

$$S_1 u_{i,j,k,l} = u_{i,j+1,k,l} - u_{i,j,k,l}$$

$$S_1 u_{i,j,k,l} = u_{i,j,k+1,l} - u_{i,j,k,l}$$

$$S_1 u_{i,j,k,l} = u_{i,j,k,l+1} - u_{i,j,k,l}$$

we can replace the analyzed continuous dynamic systems by discrete dynamic systems with distributed parameters that can be used to study in the field of four-indicator discrete signals or to numerically determine their responses.

Summary

The method of problem analysis using the non-classical operator calculus and generalized dynamical systems has been presented.

The generally accepted mathematical apparatus shows wide possibilities of creating uniform descriptions of engineering problems in various fields: continuous and discrete.

Using the formula (4), it is possible to conduct a quantitative and qualitative analysis of the problem, after identification, calibration and validation of the model, preferably using in situ tests to determine the parameters of the replacement system.

The quantities *u(x, y, z, t)* can be determined numerically, as well as using hybrid methods based on formula (4) or using discrete dynamic systems with distributed parameters approximating a given continuous dynamic system.

The dependencies obtained can be used for simulation and experimental research.

These analog models can be used to determine the failure rate, reliability of transport systems, their components and to plan their current and major repairs.

The analysis of the problem is further complicated by the interaction of individual structural elements, for example, the railway pavement, the subgrade with geosynthetics and the subsoil. The way out of this situation is to use analog and replacement circuits.

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