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**Utilization gLAB software in precise aircraft positioning  
in air transport- airborne tests in Dęblin and Mielec**

**Abstract:** This paper presents research results relate to determination aircraft coordinates using SPP method in kinematic mode. For this purpose C/A code on L1 frequency in GPS system from onboard Topcon TPS Hiper receiver were used. Numerical computations of aircraft coordinates in SPP module in gLAB software were executed. The SPP method was characterized and configuration of SPP module in gLAB software was described. In research part, two airborne tests (Mielec and Dęblin) were realized and preliminary results of positioning accuracy of aircrafts were presented. Accuracy of BLh geodetic coordinates for aircraft is better than 4 m in Mielec experiment and better than 12 m in Dęblin experiment, respectively.

**Keywords:** GPS; SPP method; MRSE; accuracy; DOP

**Introduction**

The aviation industry in Poland has been undergoing huge development and technological progress in the field of aircraft operation for GNSS satellite technology for several years. The companies manufacturing aircraft instruments create newer and better satellite receivers, designed to determine the reliable position of the aircraft based on algorithms commonly used in satellite navigation. As part of the production of receivers, special emphasis is placed on increasing the number of signals received from satellites (different constellations), increasing the frequency of signal registration, reducing the multi-track effect, calibrating and determining the hardware delay parameter for the receiver, reducing the time of first initialization of coordinates determination, the ability to receive code observations and phased on two or three carrier frequencies (see GPS system), etc. Additionally, internal software of onboard computers, coupled with satellite receivers, uses various smoothing filters in the process of navigating the user's position, which aims to increase the accuracy of the aircraft's coordinates. It is worth mentioning that software used in air navigation should enable selection of the appropriate configuration of input parameters for the GNSS sensor and calculation module. An example of such a solution is the gLAB program, which is an open-

source application and can be used as external calculation software for many tests and aeronautical research. The gLAB program was developed by a group of experts from the Technical University of Catalonia (UPC) in Spain as part of the research project N. P1081434, funded by the European Space Agency (ESA). The aim of the project was to develop computer software, aimed at determining the user's position based on satellite observations from GNSS systems. Currently, the gLAB program develops satellite observations only from the GPS system, however, in the future it will be possible to apply in the calculations of observations from the GLONASS and GALILEO systems [7]. The gLAB program is relatively easy to use and does not require manual installation by the user. All access codes for the program are freely available on the website: <http://www.gage.upc.edu> [13]. The gLAB application can be used under Windows and Linux systems, both on desktops and laptops, or operate from an external disk (e.g. a pendrive). The gLAB program has low hardware requirements: only 256 MB of operating memory, 200 MB of free disk space and 1 GHz processor. The group of users of the gLAB program focuses mainly on students, scientists and companies operating in the field of navigation and geodesy. The gLAB program can be used to determine the user's position in both static [1] and kinematic [7]. The gLAB program has two basic graphic modules, i.e. "Positioning" and "Analysis" (analysis of results). The authors of the software within the "Positioning" module proposed the use of 2 types of positioning, i.e. the Single Point Positioning (SPP) method and the Precise Point Positioning (PPP) method [9]. The article presents the preliminary results of tests from aviation tests carried out with the Cessna aircraft in Dęblin and Mielec. The position of the aircraft was determined based on GPS observations using the Kalman filter for the SPP method. The GPS navigation message and raw GPS observations in the RINEX 2.11 format were used in the calculations. The results obtained from the conducted research were presented in the work on the appropriate graphical charts.

### The algorithm for determining the aircraft coordinates for the SPP method

The SPP method is commonly used to determine user coordinates in navigation and absolute positioning. The typical positioning accuracy for the SPP method is less than 10 m, although, in the case of kinematic positioning, the final accuracy can be much lower [2]. The basic observational equation for the SPP method in the gLAB program takes the form [7, 9, 12]:

$$C1 = \rho + c \cdot (dto - dts) + Ion + Trop + Rel + TGD \quad (1)$$

where:

$C1$  - code measurement of C/A on L1 frequency in the GPS system,

$\rho$  - geometric distance of the satellite - receiver,



$(x, y, z)$  - aircraft position in a geocentric layout,

$(X_{GPS}, Y_{GPS}, Z_{GPS})$  - position of the GPS satellite in orbit,

$c$  - speed of light,

$dto$  - receiver clock error,

$dts$  - satellite clock error,

$Ion$  - ionospheric correction,

$Trop$  - tropospheric correction,

$Rel$  - relativistic correction (mainly the eccentricity of the orbit),

$TGD$  - instrumental satellite error for code measurement.

In the first stage, in the equation (1), models of physical phenomena (ionospheric and tropospheric correction for each measurement) and instrumental parameters (receiver clock error, relativistic correction and TGD group delay for each measurement) are determined. At this stage equation (1) can be written as:

$$C1+c \cdot dts-Ion-Trop-Rel-TGD= \rho+c \cdot dto \quad (2)$$

Then the equation (2) is subjected to a linearization process using the Taylor series [3]:

$$C1+c \cdot dts-Ion-Trop-Rel-TGD= (\Delta x/\rho) \cdot \sigma_x+(\Delta y/\rho) \cdot \sigma_y+(\Delta z/\rho) \cdot \sigma_z+c \cdot dto \quad (3)$$

where:

$(\sigma_x, \sigma_y, \sigma_z)$  - increments to the approximate coordinates of the receiver antenna from the RINEX observation file

$$\Delta x=x-X_{GPS},$$

$$\Delta y=y-Y_{GPS},$$

$$\Delta z=z-Z_{GPS}.$$

Equation (3) is the basic mathematical expression for the SPP method in the gLAB program and is solved by the Kalman filter in a two-stage process, as below [9]:

1) prediction process:

$$x_p=A \cdot x_0 \quad (4)$$

$$P_p=A \cdot P_0 \cdot A^T+Q_0 \quad (5)$$

where:

A- matrix of coefficients,

$x_0$ - estimated values of the a priori parameters from the previous step,

$P_0$ - estimated values of a priori covariance from the previous step,

$x_p$ - forecast of the state vector value,

$P_p$ - predicted covariance values,

$Q_0$ - a matrix of covariance of the noise process.

2) correction process:

$$K_k=P_p \cdot H^T \cdot (H \cdot P_p \cdot H^T+R)^{-1} \quad (6)$$

$$x_k=x_p+K_k \cdot (z-H \cdot x_p) \quad (7)$$

$$P_k=(I-K_k \cdot H) \cdot P_p \quad (8)$$

where:

R - measurement covariance matrix,

H – plan matrix,

$K_k$  – Kalman's strengthening matrix,

z – Kalman's strengthening matrix,

I – unit matrix

$x_k$  - set parameters a posteriori,

$P_k$  - covariance matrix of determined a posteriori parameters.

The final coordinates of the receiver antenna are obtained by adding the determined increments  $(\sigma_x, \sigma_y, \sigma_z)$  to the initial aircraft coordinates from the RINEX file  $(x, y, z)$ :

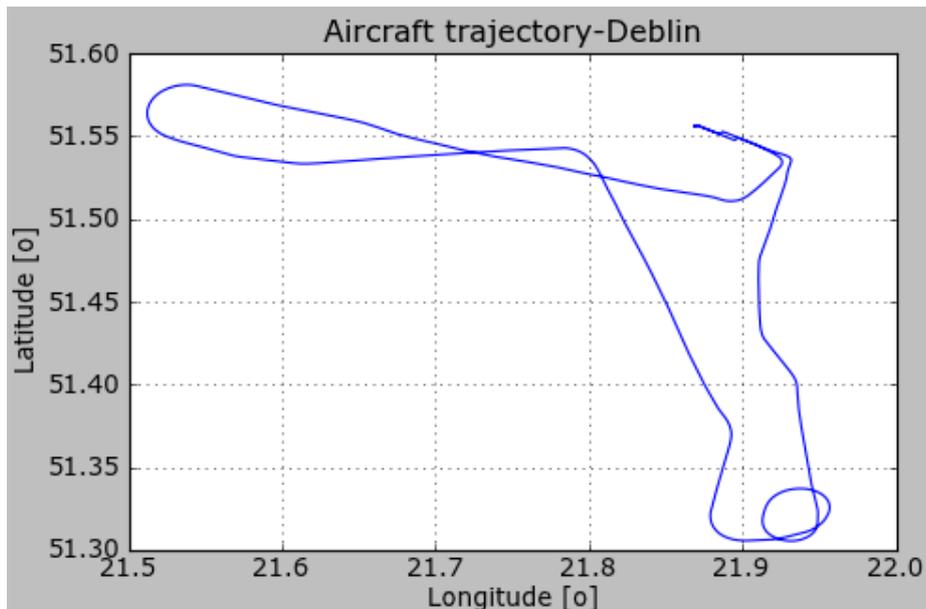
$$x_{SAM}=x+\sigma_x; y_{SAM}=y+\sigma_y; z_{SAM}=z+\sigma_z \quad (9)$$

where:

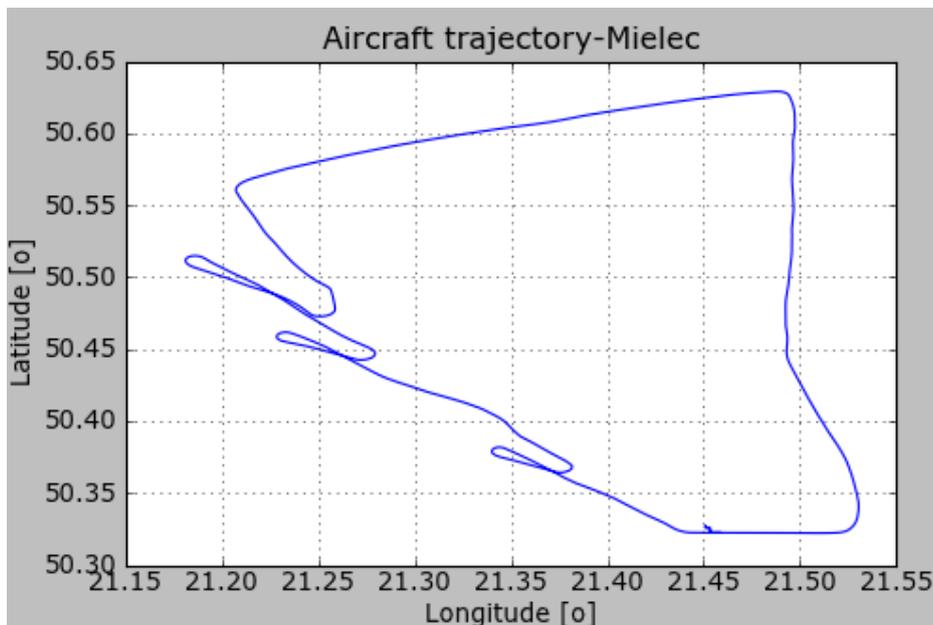
$(x_{SAM}, y_{SAM}, z_{SAM})$  - final aircraft coordinates in the XYZ geocentric system.

### Experiment and preliminary results

In the research experiment, 2 aviation tests were carried out, the aim of which was to determine the position of the aircraft and determine the accuracy of coordinates in the BLh geodetic system (B-geodetic width, L- geodetic length, h- ellipsoidal height) [8]. Aerial tests were carried out on Cessna 172 in Dęblin on 06/01/2010 [4, 5] and Mielec on 09/07/2011 [6, 7] (see Figures 1 and 2).



1. Trajectory of airplane flight in the Dęblin aviation experiment



2. The trajectory of airplane flight in the Mielec airfield experiment

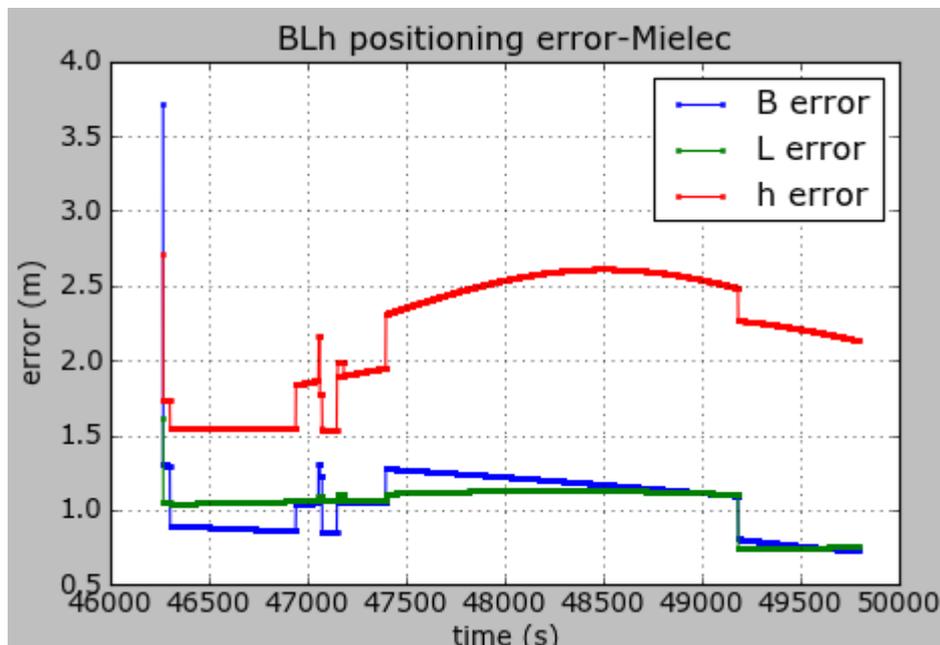
In the case of both tests in the Cessna aircraft in the cockpit, the Topcon TPS Hiper receiver antenna was installed, which recorded raw code observations (C1, P1 and P2), phase (L1 and L2) and Doppler (D1 and D2) in the GPS and GLONASS system with an interval 1 s. The receiver recorded satellite observations in the universal RINEX 2.11 format, and in the numerical calculations, only the general public C / A code was used on the L1 frequency in the GPS system. Flight time in the Dęblin experiment (06/01/2010) lasted from 09:39:03 to 10:35:03, and in the experiment Mielec (09/07/2011) was respectively from 12:50:51 to 13:49:48 (in both GPST time is given).

The development of satellite observations in the post-processing kinematic mode for the SPP method has been carried out in the gLAB program. The position of the aircraft in both tests was determined using GPS observations and parameters included in the GPS navigation message. As part of the conducted aviation experiments, 3 experiments were carried out, ie the accuracy of

individual aircraft coordinates in the BLh geodetic system was determined, the values of MRSE parameters were determined and the values of GDOP, HDOP, VDOP, and PDOP coefficients were presented.

During the calculations performed, the following configuration parameters were set for the SPP method in the gLAB program [7]:

- a set of RINEX observations: version 2.11,
- RINEX navigation data: ephemeride on-board GPS,
- source of ephemeris data and satellite clocks: ephemeride on-board GPS,
- source of ionospheric correction: GPS on-board ephemeris,
- source of instrumental errors TGD: ephemeris on-board GPS,
- initial user coordinates: based on the RINEX 2.11 observation file,
- calculation interval: 1 s,
- elevation mask: 100,
- detection of inefficient satellites: applied,
- satellite clock error correction: applied,
- taking into account the pseudo-distance time on satellite co-ordinates:
- the effect of the Earth's rotation effect on satellite coordinates: applied,
- Sagnac effect: applied,
- relativistic correction (eccentricity of the orbit): applied,
- the ionosphere model: the Klobuchara model,
- troposphere model: Simple model,
- mapping function: Simple Mapping,
- GPS observation type: L1-C / A code measurement,
- pseudo-range measurement error: 1 m,
- positioning mode: kinematic,
- calculation mode: post-processing,
- set parameters: geocentric coordinates and receiver clock error.



### 3. Positioning accuracy of the aircraft in the Mielec airfield experiment

Fig. 3 presents the accuracy of positioning of the aircraft in the BLh geodetic system for the Mielec experiment. The accuracy values for the individual BLh coordinate are marked as follows: blue indicates coordinate accuracy B, green indicates the accuracy of L coordinate, red indicates the

accuracy of ellipsoidal height  $h$ . On the basis of Fig. 3 it can be concluded that the accuracy of horizontal coordinates (geodetic width  $B$  and the geodetic length  $L$ ) is higher than the vertical coordinate  $h$ . The dispersion of the accuracy values for the  $B$  component is from 0.7 m to 3.7 m, for the  $L$  component respectively 0.8 m to 1.6 m, and for the ellipsoidal height  $h$  from 1.5 m to 2.7 m. Fig. 3, it can be observed that the accuracy of the BLh coordinates is very irregular, which is particularly evident in the mid-phase of the aircraft's flight. In the final stage of flight (intake, landing approach and touchdown), the accuracy of horizontal coordinates is higher than 1 m, and the accuracy of the vertical component  $h$  is better than 2.5 m. It is worth noting that the accuracy of BLh coordinates for all components is higher than 4 m during the entire air test.

Fig. 4 presents the values of GDOP coefficients (blue), PDOP (green), HDOP (red), VDOP (black). The values of HDOP, VDOP and PDOP coefficients were determined from dependence [10, 11]:

$$\text{HDOP}=(G(1,1)+G(2,2))^{0.5} \tag{10}$$

$$\text{VDOP}=G(3,3) \tag{11}$$

$$\text{PDOP}=(G(1,1)+G(2,2)+G(3,3))^{0.5} \tag{12}$$

$$\text{GDOP}=(G(1,1)+G(2,2)+G(3,3)+G(4,4))^{0.5} \tag{13}$$

where:

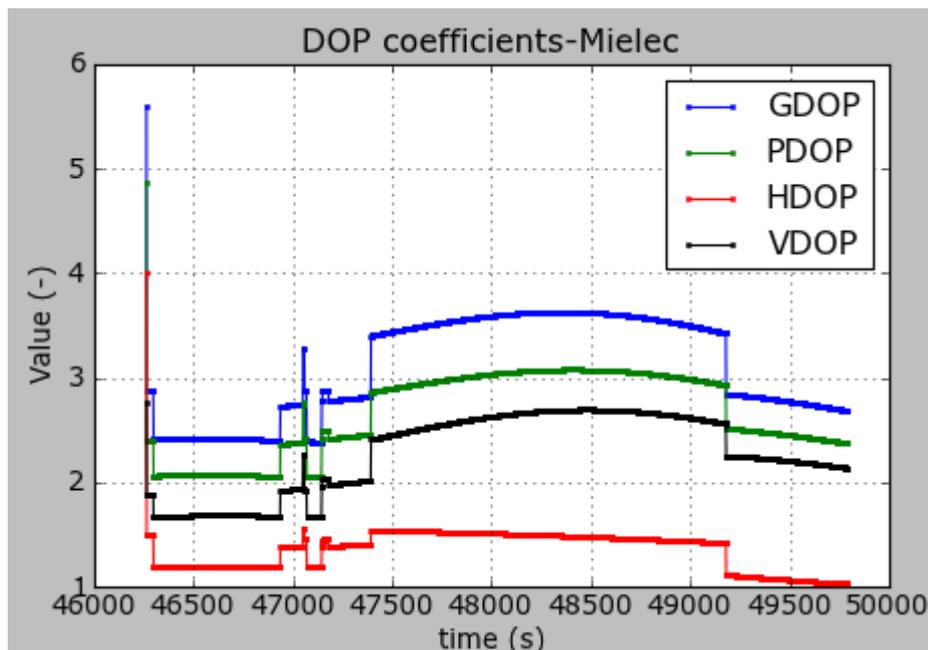
HDOP – diffusion coefficient of positions in the horizontal plane,

VDOP - vertical position blur ratio,

PDOP – dilution ratio of positions in 3D space,

GDOP – geometric blur factor,

$G=(A^T \cdot A)^{-1}$  – a full row matrix.



#### 4. DOP coefficients in the Mielec aviation experiment

In practice, the DOP coefficients take the following values from the ranges [11]:

- from 1 to 3 (very good observation conditions),
- from 4 to 5 (good observation conditions),
- from 5 to 6 (poor observation conditions),
- above 6 (unfavorable measuring conditions, do not make measurements).

In the case of the Mielec experiment, the DOP coefficients are respectively:

- HDOP from 1 to 4 (means very good and good observation conditions),
- VDOP from 1.7 to 2.8 (means very good observation conditions),

- PDOP from 2 to 4.8 (means very good and good observation conditions),
- GDOP from 2.4 to 5.6 (means very good, good and poor observation conditions).

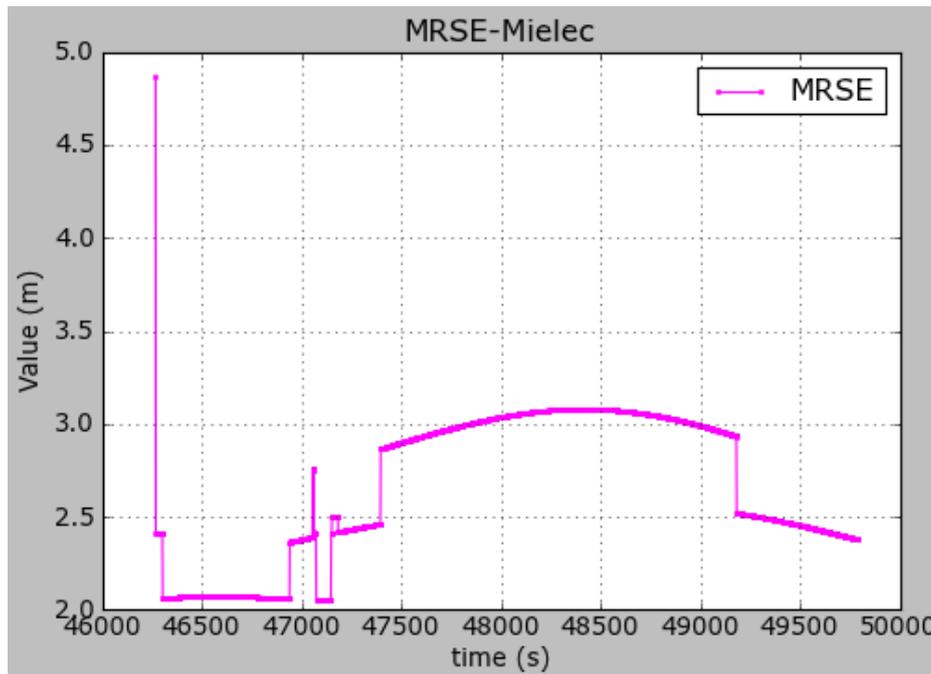
Fig. 5 presents the values of the aircraft position error parameter in 3D space - the MRSE parameter. The MRSE parameter is determined from the mathematical dependence as below [10]:

$$MRSE = (mB^2 + mL^2 + mh^2)^{0.5} \tag{14}$$

where:

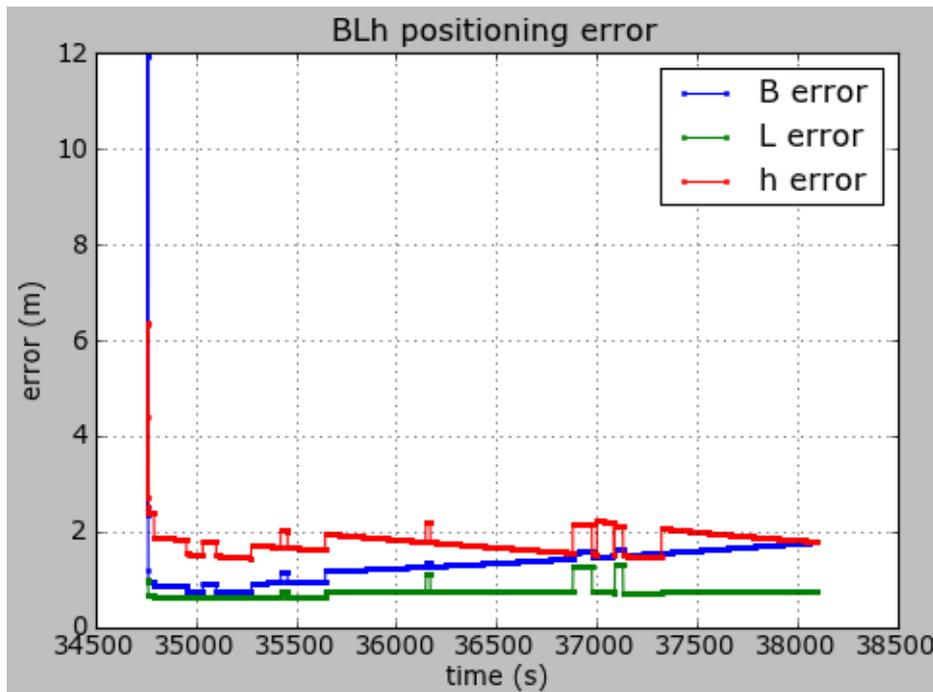
- mB – accuracy of determining the coordinate L,
- mL - accuracy of determination of the B coordinate,
- mh - accuracy of determining the coordinate h.

The value of the MRSE parameter for the aviation experiment in Mielec ranges from 2 m to 4.9 m. It should be noted that the MRSE parameter is about 3 m in the middle of the flight phase and about 2.5 m in the final phase, respectively.

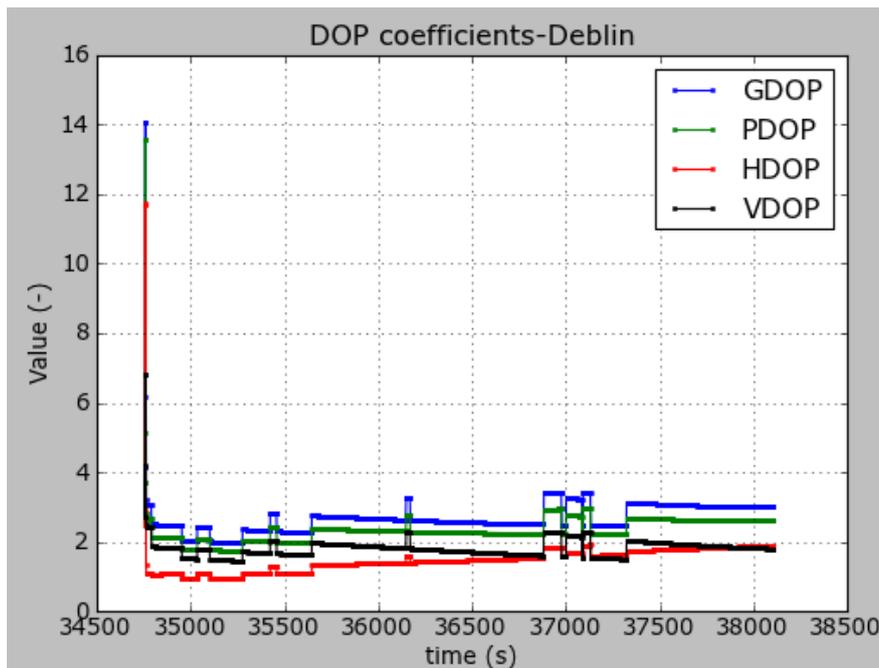


5. The value of the MRSE parameter in the Mielec aviation experiment.

Fig. 6 presents the accuracy of determining the position of the Cessna 172 aircraft in the BLh geodetic system for the Dęblin aviation experiment. The accuracy of the B coordinate was determined in blue, the accuracy of the ellipsoidal height was determined in red, and the accuracy of the horizontal coordinates (B and L) is higher than the ellipsoidal height h, but the dispersion is determined in red. the accuracy between individual coordinates has been reduced. The distribution of accuracy values for the B component is from 0.8 m to 12 m, for the L component respectively 0.7 m and 2.3 m, and for the ellipsoidal height h from 1.6 m to 6.3 m. When approaching the landing plane, the accuracy of the BLh geodetic coordinates is higher than 2 m for all 3 components. It is worth mentioning that the accuracy of BLh coordinates for all components is higher than 2.3 m for more than 99% of the duration of the flight. It should be noted that for the initial measurement periods the positioning accuracy of Cessna 172 is relatively low.



6. Positioning accuracy of an aircraft in the Dęblin aviation experiment

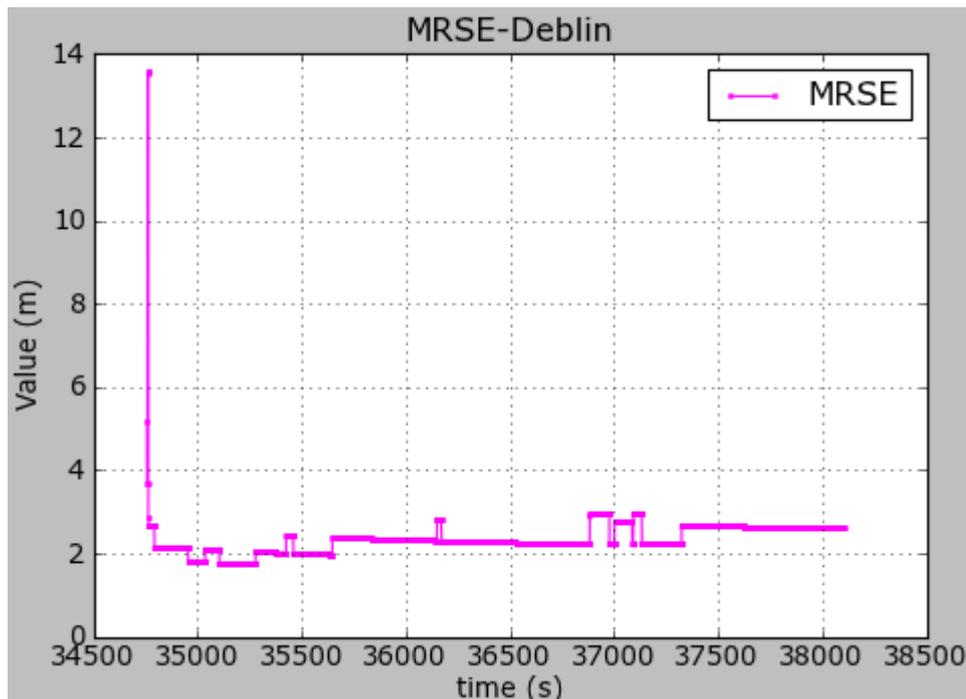


7. DOP coefficients in the Dęblin aviation experiment

Fig. 7 presents the values of GDOP coefficients (blue), PDOP (green), HDOP (red) and VDOP (black). In the case of the Dęblin experiment, the DOP coefficients are respectively:

- HDOP from 1 to 11.9 (means very good and relatively unfavorable observation conditions),
- VDOP from 1.7 to 6.7 (during the experiment the observation conditions were very good and poor),
- PDOP from 1.8 to 13.8 (during the experiment the observation conditions were very good and unfavorable),
- GDOP from 2 to 14 (during the experiment the observation conditions were very good and unfavorable).

In comparison with the HDOP, VDOP, GDOP and PDOP coefficients from the Mielec experiment, the precision of the DOP coefficients in the Dęblin experiment deteriorated.



8. The value of the MRSE parameter in the Dęblin aviation experiment

Fig. 8 shows the values of the MRSE parameter for the aviation experiment in Dęblin with the Cessna 172 aircraft. The value of the MRSE parameter for the aviation experiment in Dęblin is from 1.9 m to 13.8 m. It should be noted that the MRSE parameter is about 3 in the middle and final flight phase. It is worth adding that for the initial measuring periods the value of the MRSE parameter is relatively low.

## Conclusions

The article presents the possibility of using the SPP method in the kinematic mode in the gLAB program to determine the position of the aircraft in post-processing. The SPP method was presented and characterized along with a description of the full algorithm for determining the position of the aircraft in the kinematic mode. Airplane coordinate calculations have been carried out for C / A coding observations in the GPS system, obtained from the Topcon TPS Hiper mobile receiver. The aviation experiments were carried out with the Cessna 172 aircraft on 01.06.2010 in and around the city of Dęblin and on 07/09/2011 near the Mielec airport. The following conclusions were drawn from the conducted research:

- the accuracy of determining the geodetic width B in the Mielec experiment was from 0.7 m to 3.7 m, while in the Dęblin experiment, respectively from 0.8 m to 12 m;
- the accuracy of determining the geodetic length L in the Mielec experiment was from 0.8 m to 1.6 m, whereas in the Dęblin experiment from 0.7 m to 2.3 m respectively;
- the accuracy of the ellipsoidal height h in the Mielec experiment was from 1.5 m to 2.7 m, while in the Dęblin experiment from 1.6 m to 6.3 m respectively;
- values of HDOP, VDOP, GDOP and PDOP coefficients in the Mielec experiment are characterized by very good, good and poor observation conditions, whereas in the Dęblin experiment respectively very good, good and unfavorable measurement conditions;
- the obtained positioning accuracy of aircraft in 3D space (MRSE parameter) is from 2 m to 4.9 m for the Mielec experiment, and respectively from 1.9 to 13.8 m for the Dęblin experiment.

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