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**Analytical design method of reverse curves**

**Abstract:** The paper deals with a new approach relating to the design of the region of railway track direction alteration adapted to Mobile Satellite Measurements technique. The method may be particularly useful in situations when both the straights of the route directions cannot be connected in an elementary way using a circular arc with transition curves; this also refers to the application of a compound curve. Thus the only solution becomes the application of two circular arcs of opposite curvature signs to the geometric system, i.e. the use of an inverse curve. The solution of the design problem takes advantage of a mathematical notation and concentrates on the determination of universal equations describing the entire geometric system. This is a sequential operation involving successive parts of the system. The presented method has been illustrated by appropriate calculation examples.

**Keywords:** Track geometrical layout; Designing; Computer Aided Design

**Introduction**

Inspiration to take up this issue is undoubtedly the appearance of new technology - used in the design and exploitation of railways satellite measurements *GPS*. Global Positioning System *GPS* [2,15-17] allows the setting coordinates of points in a single, three-dimensional reference system *WGS 84*, which beginning is located in the center of mass of the Earth. Designated by means of measurements *GPS* coordinates ellipsoid are converted by mapping the Gauss-Krüger in order to obtain a coordinates flat rectangular [17]. These coordinates make up in Poland the so-called PL-2000 system, which is one of the elements of national spatial reference system.

Ability to obtain adequate performance measurement in the railway track gives developed by a scientific team Gdansk University of Technology and the Naval Academy / Academy of Gdynia Maritime Mobile satellite surveying technique [8-10], which consists of a tour of the test section of the route with antennas installed on a moving rail vehicle. Mobile Satellite Measurements allow to determine the coordinates of the existing railway line in Cartesian [9,11,12]. In such a situation, it becomes obvious that in that arrangement should be determined also coordinates the newly designed center of the track, designed to set a route in the area. The resulting very high precision the coordinates in the horizontal plane (with an error of a few millimeters) leads to the development of new methods of design of track geometrical layout [4-6] and new programs of computer support for such projects [7], adapted to the techniques of satellite measurements.

Designing geometrical layouts of railways usually takes place in relation to the existing state; it is therefore design axis adjustment of the track. In case of the region change the direction of the route it involves the correction of the radius of the arc and the type and length of transition curves to the new geometrical arrangement as beneficial as possible from the point of view of the kinematics of rail traffic and the cost of maintaining the track. At the same time the position of the track in a horizontal plane should not deviate too much from the existing position.

The key issue is to determine geometrical parameters, which ensure the fulfillment of these conditions. The designation of these parameters requires consideration of many design options and to choose the most beneficial. To generate variants a new method of calculating coordinates is needed, adapted to the techniques of satellite measurements. Only after obtaining a result of this procedure, the appropriate values of geometrical parameters can be used in a rational manner any of the commercial computer programs supporting design.

In some situations to connect with each of the main directions of the route cannot be used model system: transition curve- a circular curve - transition curve [5], nor solve the problem applying a compound curve [4]. It remains the introduction to the geometric arrangement of the two inverse curves. Inverse curves can also be a beneficial alternative in case of the need to avoid an field obstacle. Description of the design procedure of inverse curves is the subject of this work. This allows direct solution for interconnecting the main directions that are not parallel. In case of connecting parallel tracks, it is required to correct the obtained numerical solutions. Presented concept of how to design the area to change the direction of the route runs - as in other developed methods - to obtain analytical solution using the appropriate mathematical formulas, so the most friendly in the practical application.

### General assumptions

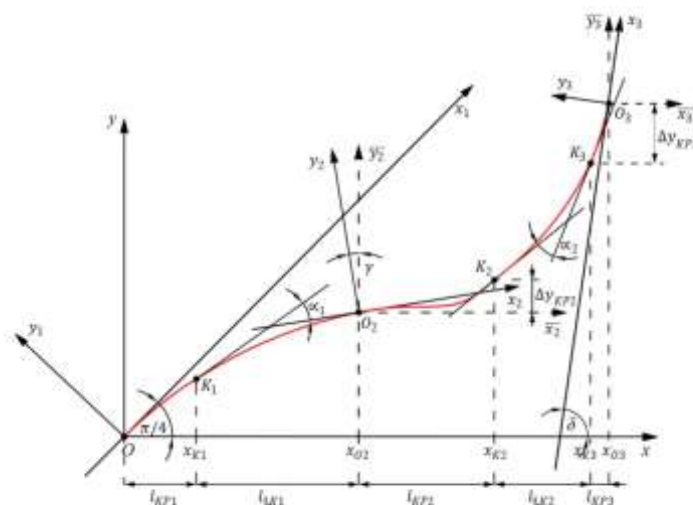
We have equations of directions of the main routes in layout PL-2000:

$$\text{Line 1} \quad X_1 = A_1 + B_1 Y$$

$$\text{Line 2} \quad X_2 = A_2 + B_2 Y$$

These lines have values of slope coefficient  $B_1$  and  $B_2$  very close to each other and intersect in a remote point (they can be also parallel to each other). Such a situation justifies the combination of two simple inverse curve. The justification for the connection by inverse curves can also be a desire to keep the track within the embankment [1].

Proper design will be performed in a suitable local coordinate system  $x, y$  ( $LUW$ ) – figure 1. This layout arises from the beginning of the new shift system to the point  $O(Y_0, X_0)$  on *Line 1* and make a whole rotation of angle  $\beta$ . The main issue becomes at the same time obtaining appropriate slopes *Lines 1 i 2* in system *UCS*.



1. Examined geometrical layout in the local coordinate system

We assume that in layout  $x, y$  *Line 1* will cross through the origin of the system with the same angle of inclination equal to  $\frac{\pi}{4}$ . The adoption of such inclination guaranteed to operate with positive values designated horizontal ordinates and provides the ability to store a mathematical of the curves in the form of an explicit function  $y(x)$ . Because *Line 2* has a similar slope to the slope *Line 1*, certainly it will be located within the range  $\left(0, \frac{\pi}{2}\right)$ , closer to the middle of the range (i.e. slope *Line 1*) than its borders. The angle of rotation  $\beta$  is determined by following formula:

$$\beta = \varphi_1 - \frac{\pi}{4} \quad (1)$$

when  $\varphi_1 = \text{atan } B_1$  for  $B_1 > 0$  and  $\varphi_1 = \text{atan } B_1 + \pi$  for  $B_1 < 0$ . Obtaining with above formula positive value of angle  $\beta$  means the rotation of layout to the left, while the for the negative value - rotation of layout to the right.

After the adoption of the coordinate point  $O(Y_0, X_0)$  on *Line 1* and determining the angle of rotation  $\beta$  we make transformation of *Lines 1* and *2* the local coordinate system  $x, y$ . The whole examined geometrical layout the UCS is shown in Figure 1.

**Tab. 1.** Characteristics of essential points of the geometrical layout

Punkt	Odcięta $x$ [m]	Rzędna $y$ [m]	Nachylenie stycznej $s$
$O$	0	0	1,000
$K_1$	65,496	61,680	0,834
$O_2$	193,254	139,202	0,417
$K_2$	344,580	191,579	0,386
$K_3$	492,704	274,569	0,764
$O_3$	546,613	319,204	0,861

The position of any point on the route in the local coordinate system  $x, y$  can be determined by formulas [14]:

$$x = (Y - Y_0) \cos \beta + (X - X_0) \sin \beta \quad (2)$$

$$y = -(Y - Y_0) \sin \beta + (X - X_0) \cos \beta \quad (3)$$

Angle  $\delta$ , determining the slope *Line 2* to axis  $x$  in the UCS, is determined by the formula

$$\delta = \varphi_2 - \beta \quad (4)$$

when  $\varphi_2 = \text{atan } B_2$  for  $B_2 > 0$  and  $\varphi_2 = \text{atan } B_2 + \pi$  for  $B_2 < 0$ .

### The selection of parameters designed geometrical layout

Designed geometric layout, linking *Line 1* with *Line 2* to the local coordinates  $x, y$ , consists of the following components (Fig. 1):

- a first transition curve (*KP1*), a particular type, length  $l_1$ ,
- a first circular curve (*ŁK 1*) with a radius  $R_1$  and length  $l_{ŁK1}$ ,
- second transition curve (*KP2*), a specific type, length  $l_2$ ,
- a second circular curve (*ŁK2*) with radius  $R_2$  and undefined length  $l_{ŁK2}$ ,

- third transition curve (*KP3*), a specific type, length  $l_3$ .

Values  $l_1$ ,  $R_1$ ,  $l_2$ ,  $R_2$  i  $l_3$  result from the speed analysis conducted for the proposed system, the value  $l_{LK1}$  is in principle optional, and  $l_{LK2}$  a result value, closing the entire system. However, you should be aware of that, the task can be resolved only with the appropriate configuration of above parameters. The key is characteristics of appearing curvature. Curve *KP1* and arc *LK1* they have a negative curvature, and the arc *LK2* and curve *KP3* – a positive curvature. This results that on the curve *KP1* and arc *LK1* the angle of the tangent to the  $x$ -axis in the UCS decreases, and on the arc *LK2* and curve *KP3* growing and it depends on mutual location of *Line1* and *2* (**Fig. 1**). The only element on which there are different character of curvature, is curve *KP2*.

### Transition curve *KP1*

Ordinates of the transition curve *KP1* we designate in the auxiliary coordinate system  $x_1, y_1$  (*PUWI*) – figure 1. The procedure is following:

- specify the type of transition,
- accept the length of the curve  $l_1$  (measured on the same curve) and the radius  $R_1$  adjoining to a circular arc *LK1*,
- otrzymujemy równania parametryczne:  $x_1(l), y_1(l)$ ,  $l \in \langle 0, l_1 \rangle$ .

Because for typical transition curves connecting line with circular arc (where the curvature is determined by equal number of boundary conditions [3])  $|\theta(l_1)| = \frac{l_1}{2R_1}$  [rad], tangent value at the end of the transition curve *KP1* is defined by the formula  $y_1'(l_1) = -\tan\left(\frac{l_1}{2R_1}\right)$ .

The next step is the transformation of the curve *KP1* to a local coordinate system  $x, y$  (**Fig. 1**). This is done through the axis rotation of layout *PUWI* to the right with angle  $\frac{\pi}{4}$ . We get the following parametric equation:

$$x(l) = x_1(l) \cos \frac{\pi}{4} - y_1(l) \sin \frac{\pi}{4} \quad (5)$$

$$y(l) = x_1(l) \sin \frac{\pi}{4} + y_1(l) \cos \frac{\pi}{4}, \quad l \in \langle 0, l_1 \rangle \quad (6)$$

Substituting equations (5) and (6) with the final value of the parameter  $l$  (i.e.  $l = l_1$ ), we obtain the coordinates of the end of the curve *KP1* (i.e. point  $K_1$ ):  $x_{K1} = l_{KP1}$  i  $y_{K1}$ . The value of the tangent  $s_{K1}$  at the point  $K_1$  total

$$s_{K1} = \tan\left(-\frac{l_1}{2R_1} + \frac{\pi}{4}\right) \quad (7)$$

### The circular arc *LK1*

Diagram showing the location of the circular arc *LK1* is shown in Figure 2. We accept length of the circular arc  $l_{R1}$  (measured along the arc). We determine the coordinates of the point  $S_1(x_{S1}, y_{S1})$  – the middle of arc *LK1*.

$$x_{S1} = x_{K1} + \frac{s_{K1}}{\sqrt{1+s_{K1}^2}} R_1 \quad (8)$$

$$y_{S1} = y_{K1} - \frac{1}{\sqrt{1+s_{K1}^2}} R_1 \quad (9)$$

The equation of the circular arc  $LKI$  is:

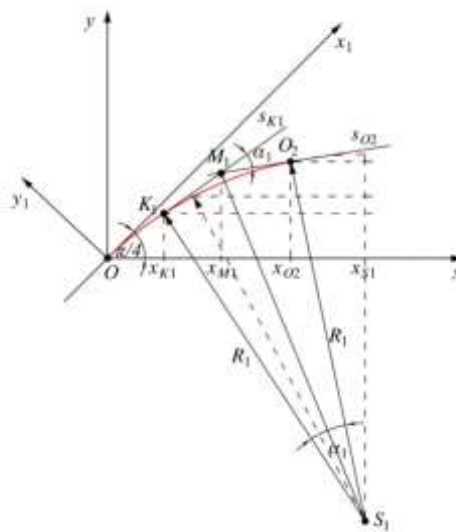
$$y(x)_{LK1} = y_{S1} + [R_1^2 - (x_{S1} - x)^2]^{\frac{1}{2}}, \quad x \in \langle x_{K1}, x_{O2} \rangle \quad (10)$$

The angle of return arc tangential  $LKI$  total

$$\alpha_1 = \frac{l_{R1}}{R_1} \quad (11)$$

And slope  $s_{O2}$  of tangent to arc  $LKI$  at her end, i.a. at the point  $O_2$

$$s_{O2} = \tan(\text{atan } s_{K1} - \alpha_1) \quad (12)$$



## 2. Diagram showing the location the circular curve $LKI$

To determine the coordinates of the point  $O_2$  – the end of the circular arc  $LKI$ , you must first determine the coordinates of the point  $M_1$  (fig. 2); it finally total

$$x_{O2} = x_{K1} + \tan \frac{\alpha_1}{2} \left( \frac{1}{\sqrt{1+s_{K1}^2}} + \frac{1}{\sqrt{1+s_{O2}^2}} \right) R_1 \quad (13)$$

$$y_{O2} = y_{K1} + \tan \frac{\alpha_1}{2} \left( \frac{s_{K1}}{\sqrt{1+s_{K1}^2}} + \frac{s_{O2}}{\sqrt{1+s_{O2}^2}} \right) R_1 \quad (14)$$

### Transition curve $KP2$

Diagram showing the location of the transition curve  $KP2$  is shown in Figure 3. The ordinates of transition  $KP2$  we set in the auxiliary coordinate system  $x_2, y_2$  ( $PUW2$ ). Transition curve  $KP2$  combines inverse curves with radius  $R_1$  i  $R_2$ . The procedure is following:

- we assume a linear or non-linear course of the curvature,

- we assume the length of the curve  $l_2$  (measured on the same curve) and ray  $R_2$  the adjoining the circular arc  $LK2$ ,
- we determine the parametric equations:  $x_2(l), y_2(l), l \in \langle 0, l_2 \rangle$ .

Exemplary solutions for the linear and non-linear distribution of the curvature of the curve, are shown in [3].

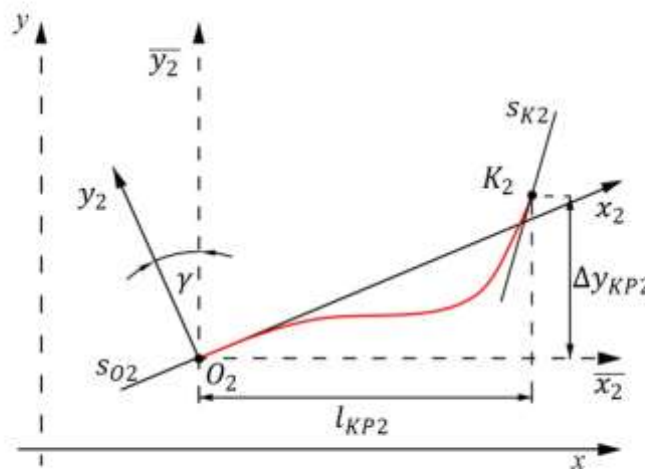
The next step is the transformation of curve  $KP2$  to the auxiliary coordinate system  $\bar{x}_2, \bar{y}_2$  ( $\overline{PUW2}$ ), as shown in Figure 3. Location of the system determines the angle  $\gamma = \text{atan } s_{O2}$ . If slope  $s_{O2} = \tan(\text{atan } s_{K1} - \alpha_1) > 0$ , then you should make the rotation of system  $PUW2$  right, and if the slope  $s_{O2} < 0$ , the system  $PUW2$  should be rotated to the left. Because  $\text{tangle } \gamma = |\text{atan } s_{O2}| \in \langle 0, \frac{\pi}{2} \rangle$ , we get

$$x = x_{O2} + \bar{x}_2(l) = x_{O2} + x_2(l) \cos \gamma \mp y_2(l) \sin \gamma \quad (15)$$

$$y = y_{O2} + \bar{y}_2(l) = y_{O2} \pm x_2(l) \sin \gamma + y_2(l) \cos \gamma, \quad l \in \langle 0, l_2 \rangle \quad (16)$$

From formulas (15) and (16) we can determine the value of  $l_{KP2}$  and  $\Delta y_{KP2}$  (fig. 3), then the coordinates of the point  $K_2$ . Taking into account the angle of inclination of the tangent  $\theta(l_2)$  at the end of the curve in the system  $PUW2$  we receive tangent value  $s_{K2}$  at the point  $K_2$  in layout  $\overline{PUW2}$ .

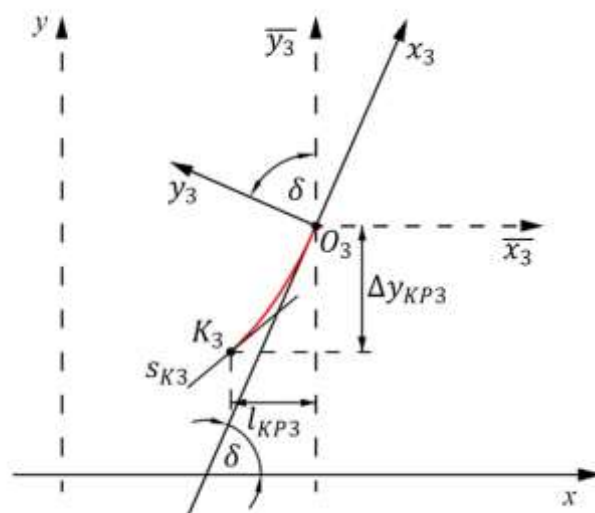
$$s_{K2} = \tan[\theta(l_2) \pm \gamma] \quad (17)$$



3. Diagram showing the location of the transition curve  $KP2$

**Transition curve  $KP3$**

To be able to determine the unknown in this phase of position of the end of the arc  $LK2$  must be addressed first transition curve  $KP3$ . This will determine the key terms of the arc tangent value  $LK2$   $s_{K3}$  at its end. Diagram showing the location of the transition curve  $KP3$  in the auxiliary coordinate system  $x_3, y_3$  ( $\overline{PUW3}$ ) is shown in figure 4.



4. Diagram showing the location of the transition curve  $KP3$

The procedure is following:

- we specify the type of transition,
- we assume the length of the curve  $l_3$  (measured along the curve),
- we obtain parametric equation:  $x_3(l), y_3(l)$ ,  $l \in \langle -l_3, 0 \rangle$ .

The next step is the transformation of the of transition  $KP3$  to the auxiliary coordinate system  $\bar{x}_3, \bar{y}_3$  ( $P\bar{U}\bar{W}3$ ), kwhich axes are parallel to the coordinate system  $x, y$ . We get it by turning the system  $P\bar{U}\bar{W}3$  to the right with tangle  $\delta$ . Parametric equations of the curve  $KP3$  in system  $P\bar{U}\bar{W}3$  are following:

$$\bar{x}_3(l) = x_3(l) \cos \delta - y_3(l) \sin \delta \quad (18)$$

$$\bar{y}_3(l) = x_3(l) \sin \delta + y_3(l) \cos \delta, \quad l \in \langle -l_3, 0 \rangle \quad (19)$$

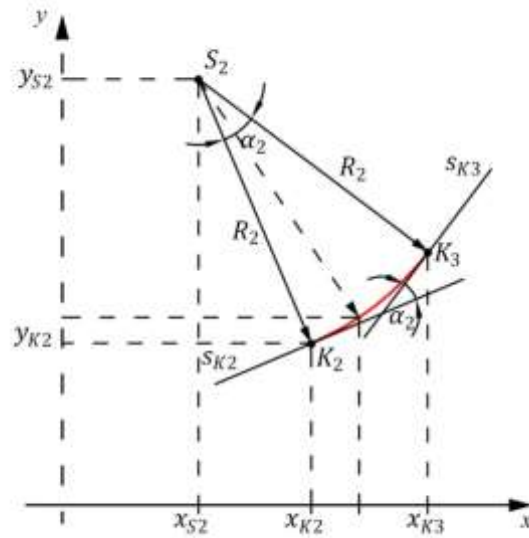
Inserting to equations (18) and (19) the final value of the parameter  $l$  (tj.  $l = -l_3$ ), we get the value  $l_{KP3}$  and  $\Delta y_{KP3}$ . The value of tangent  $s_{K3}$  at the point  $K_3$  in layout  $P\bar{U}\bar{W}3$  (and in UCS) is

$$s_{K3} = \tan \left( -\frac{l_3}{2R_2} + \delta \right) \quad (20)$$

Writing equations of curve  $KP3$  in the local coordinate system requires the designation of a point  $O_3(x_{O3}, y_{O3})$ .

### Circular arc $LK2$

A diagram illustrating the position the circular curve  $LK2$  shown in Figure 5. At this stage of the design are known, in addition to the radius  $R_2$ , the coordinates of the starting point  $K_2(x_{K2}, y_{K2})$  and the value of the tangent at the beginning  $s_{K2}$  and end  $s_{K3}$  of arc  $LK2$  (position of the end of the curve, so the coordinates of the end point  $K_3(x_{K3}, y_{K3})$  jest is unknown). First we determine the coordinates of the point  $S_2(x_{S2}, y_{S2})$  – the middle of arc  $LK2$ .



5. Diagram showing the location the circular curve  $LK2$

$$x_{S2} = x_{K2} - \frac{s_{K2}}{\sqrt{1+s_{K2}^2}} R_2 \quad (21)$$

$$y_{S2} = y_{K2} + \frac{1}{\sqrt{1+s_{K2}^2}} R_2 \quad (22)$$

The equation of the circular curve  $LK2$  is:

$$y(x)_{LK2} = y_{S2} - [R_2^2 - (x - x_{S2})^2]^{\frac{1}{2}}, \quad x \in (x_{K2}, x_{K3}) \quad (23)$$

With condition

$$y'(x_{K3})_{LK2} = -\frac{x_{K3} - x_{S2}}{[R_2^2 - (x_{K3} - x_{S2})^2]^{\frac{1}{2}}} = s_{K3}$$

we can determine  $x_{K3}$ , and then  $y_{K3}$ .

$$x_{K3} = x_{S2} + \frac{s_{K3}}{\sqrt{1+s_{K3}^2}} R_2 \quad (24)$$

$$y_{K3} = y_{S2} - \frac{1}{\sqrt{1+s_{K3}^2}} R_2 \quad (25)$$

Length of the projection curve  $LK2$  on axis  $x$  is

$$l_{LK2} = x_{K3} - x_{K2} \quad (26)$$

and the angle of return tangents

$$\alpha_2 = |\text{atan } s_{K3} - \text{atan } s_{K2}| \quad (27)$$



It follows directly the length of the circular curve  $LK2$  (measured on the curve)

$$l_{R2} = \alpha_2 R_2 \quad (28)$$

Knowing the location of a point  $K_3(x_{K3}, y_{K3})$  allows to specify coordinates of the end point  $O_3(x_{O3}, y_{O3})$  – figure 4.

$$x_{O3} = x_{K3} + l_{KP3} \quad (29)$$

$$y_{O3} = y_{K3} + \Delta y_{KP2} \quad (30)$$

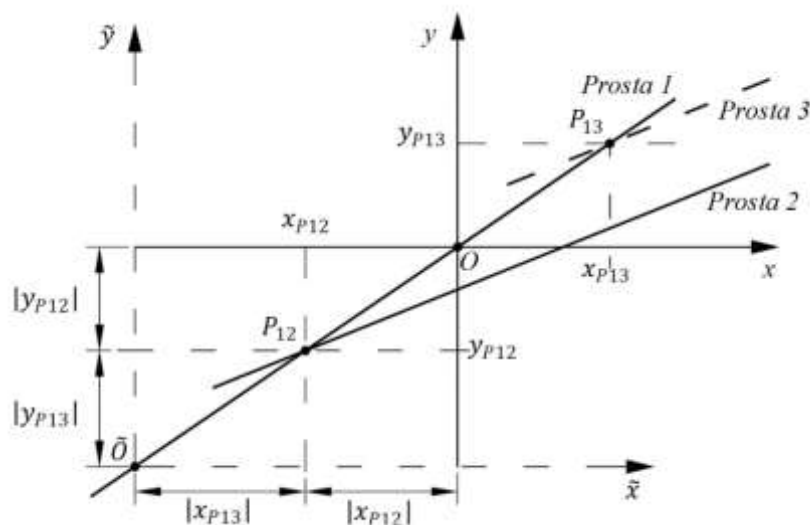
and parametric equations of the transition curve  $KP3$ :

$$x(l) = x_{O3} + \bar{x}_3(l) = x_{O3} + x_3(l) \cos \delta - y_3(l) \sin \delta \quad (31)$$

$$y(l) = y_{O3} + \bar{y}_3(l) = x_{O2} + x_3(l) \sin \delta + y_3(l) \cos \delta, \quad l \in \langle -l_3, 0 \rangle \quad (32)$$

### Designing the proper location of the beginning of the local coordinate system

To be able to move designed geometrical layout with UCS to the 2000 system, adjust the position of the starting point on *Line 1*. In the general case, the point  $O_3(x_{O3}, y_{O3})$ , marks the end of the system, it will not lay on *Line 2*, but in parallel thereto *Line 3* in formula  $y_{(3)} = a_3 + b_2x$  (fig. 6).



### 6. The idea of how to correct the coordinates of the beginning of UCS

As previously mentioned, the presented method allows a direct solution to the problem in the case of a merger with another non-parallel directions. It is possible then to determine corrected coordinates of the beginning of the UCS. To do this, calculate the coordinates of the points of intersection *Line 1* with *Line 2* and with *Line 3*. *Line 1* intersects with *Line 2* at the point  $P_{12}$

$$x_{P12} = \frac{a_2}{1-b_2} = y_{P12} \quad (33)$$

with *Line 2* in point  $P_{13}$

$$x_{P_{13}} = \frac{y_{O_3} - b_2 x_{O_3}}{1 - b_2} = y_{P_{13}} \quad (34)$$

Our task is to shift the beginning of the UCS layout (point  $O$ ) among *Line 1* to a new location at the point  $\tilde{O}$ , that in layout  $\tilde{x}, \tilde{y}$  obtains  $\tilde{x}_{P_{12}} = x_{P_{13}}$  and of course  $\tilde{y}_{P_{12}} = y_{P_{13}}$  (fig. 6). This leads to the following general formula:

$$x_{\tilde{O}} = y_{\tilde{O}} = x_{P_{12}} - x_{P_{13}} \quad (35)$$

Corrected beginning of the local coordinate system 2000 has the coordinates:

$$Y_{\tilde{O}} = Y_O + x_{\tilde{O}} \cos \beta - y_{\tilde{O}} \sin \beta \quad (36)$$

$$X_{\tilde{O}} = X_O + x_{\tilde{O}} \sin \beta + y_{\tilde{O}} \cos \beta \quad (37)$$

Transfer of designed geometrical layout with the UCS 2000 system is performed using the following assumptions [14]:

$$Y = Y_{\tilde{O}} + x \cos \beta - y \sin \beta \quad (38)$$

$$X = X_{\tilde{O}} + x \sin \beta + y \cos \beta \quad (39)$$

In case of joining together the parallel directions you are required to correct the solutions obtained by numerical determination of geometrical parameters corresponding to the present distance between track.

### Calculation example

The opportunities offered by the presented design method shows the calculation example, which merged the two directions of route of similar slope coefficients in the coordinate system PL-2000. The basic assumption was to adopt train speed  $v = 90$  km/h.

$$\text{Line 1 } X_1 = 27878029,36485 - 3,365805196 Y$$

$$\varphi_1 = 1,859595714 \text{ rad}$$

$$\beta = 1,074197551 \text{ rad}$$

$$\text{Line 2 } X_2 = 35851110,27727 - 4,592528563 Y$$

$$\varphi_2 = 1,785194693 \text{ rad}$$

$$\text{Established starting point } Y_O = 6498745,04911 \text{ m}$$

$$X_O = 6004519,50986 \text{ m}$$

Design data

$$v = 90 \text{ km/h}$$

$$l_1 = 90 \text{ m} \quad \text{clothoid}$$

$$R_1 = 500 \text{ m} \quad l_{R1} = 150 \text{ m} \quad h_1 = 100 \text{ mm}$$

$$l_2 = 160 \text{ m} \quad \text{linear curve}$$

$$R_2 = 600 \text{ m} \quad (l_{R2} = 170,359 \text{ m}) \quad h_2 = 70 \text{ mm}$$

$$l_3 = 70 \text{ m} \quad \text{clothoid}$$

when:  $h_1, h_2$  – cant value in curves, respectively,  $\text{ŁK1}$  i  $\text{ŁK2}$ .

Correction of the position of the beginning of the UCS

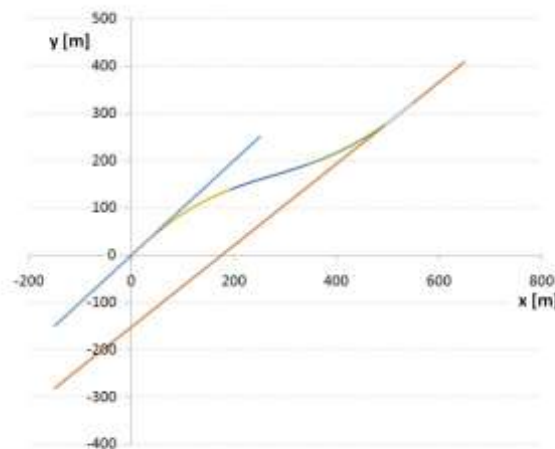
$$Y_{\tilde{o}} = 6,499053,754 \text{ m}$$

$$X_{\tilde{o}} = 6003480,469 \text{ m}$$

$$\text{Line 1 } y_{(1)} = x$$

$$\text{Line 2 } y_{(2)} = -257,909 + 0,86126 x$$

The practical usefulness of the presented solution (Fig. 7) cannot raise any doubts. From the point of view of the same calculation methodology some problems creates combining parallel lines, since it can then be made directly correcting the position of the local coordinate system. Computer character of calculations using the presented algorithm can, however, easily resolve this issue. It should also be noted that the design you have to sometimes get used to bypass an obstacle field and inverse curves can then be an alternative even for elementary geometric problem, which is the joining together of two straight lines using the circular curve.



## 7. Visualization of designed geometrical layout

### Summary

- Application of Mobile Satellite Measurements of Antennas installed on a moving rail vehicle, enables you to restore the position of the axis of the tracks in the absolute frame of reference. This creates completely new possibilities for shaping geometric railway tracks. In this situation there is a need to adapt the algorithms to design new technical possibilities.
- This paper presents another (as described in the papers [4-6]) design method for region change the direction of the railway, adapted to the technique of Mobile Satellite Measurements. This method can be particularly useful when both the simple directions of the route cannot be combined in a elementary way, using a circular curve with transition curves. This situation occurs when combined together lines have simple slope coefficient values very close to each other and intersect in a remote point (they can also be parallel to each other). The only solution then becomes the introduction to the geometrical layout two circular curves of opposite sign of curvature, that is, applying the inverse curve.
- Presented concept of a method for designing the change of route area results in an analytical solution, using suitable mathematical formulas, and therefore the most favourable in practical use. The design procedure is universal in nature and creates the possibility of taking any lengths and rays of the curves and the diversification of the type and length of the used transition curves.

**Source materials**

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